Should I open to forecast? Implications from A Multi-country Unobserved Components Model with Sparse Factor Stochastic Volatility *

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May 2023

Abstract: In this paper, we assess whether and when multi-country studies pay off for forecasting inflation and output growth. Factor stochastic volatility is adopted to allow for cross-country linkages and model economies jointly. We estimate factors and rely on the post-processing, rather than expert judgement, to obtain an estimate for the number of factors. This is different from most existing two-step approach in the factor literature. Our approach is then used to extend the existing unobserved components model which assumes 34 economies are independent. The results suggest that allowing for cross-country linkages yields inflation and output growth forecasts that are highly competitive to estimating economies independently. Zooming into the forecast performance over time reveals that allowing for cross-country linkages is of particular importance when interest centers on forecasting periods of uncertainty. Another key finding is that the estimated global factors are correlated with the domestic business cycle. We interpret this as that part of the variation captured in global factors reflects a global business cycle.

JEL classification: F44, C11, C55

Keywords: factor stochastic volatility, sparsification, unobserved components models, global business cycle

*I am extremely grateful to Gary Koop and Aubrey Poon for the valuable guidance and the feedback throughout the research and writing process. I am also indebted to Joshua Chan, Sharada Davidson and Saeed Zaman for helpful comments.

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1 Introduction

Since the seminal work by Stock and Watson (2007), the unobserved components (UC) model with stochastic volatility (SV) is commonly used for modeling latent state vectors. These latent state vectors can be interpreted as long-run equilibrium levels and the UC model has enjoyed great popularity. Surprisingly, to the best of our knowledge, existing literature imposes an independent assumption across economies. However, the studies of global macroeconomic developments argue that national macroeconomic developments depend on international conditions. The dependence holds for both the real business cycle (see Kose et al., 2003) and inflation (see Ciccarelli and Mojon, 2010).

Investigating whether and when allowing for cross-country linkages pay off for inflation and output forecasting is the key objective of the present paper. Building on recent advances in econometrics, we adopt the factor stochastic volatility (FSV) to allow for cross-country linkages and model economies jointly. To avoid omitting some potentially important factors, we adopt shrinkage techniques which use the sparsification on factor loadings and rely on the post-processing to obtain an estimate for the number of factors.

From an empirical standpoint it is necessary to investigate how these techniques perform overall and over time. We show this by carrying out a thorough forecasting experiment involving 34 economies. The economies considered include 23 Advanced Economies (AEs) and 11 Emerging Market Economies (EMEs). We include two variables in each economy (quarterly CPI inflation and output growth).

Our results show that the cross-country linkages techniques yield forecasts that are competitive to the ones obtained from estimating economies independently. When the focus is on forecasting periods of uncertainty, we find these techniques can provide great improvements.

Additionally, we find the slope of the Phillips curve becomes lower when using the FSV. This seems to be relevant to the debate about the flattening of the Phillips curve. However, we think one needs to interpret this lower value with some care. By checking the correlation between the estimated global inflation factors and the domestic business cycle, we find they are positively correlated. In this sense, we interpret this as that part of the variation captured in global inflation factors reflects a global business cycle. Adding these factors can reduce the omitted variable bias.

This paper is organized as follows. Section 2 reviews the related empirical literature and explains our contributions. In Section 3, we first discuss the UC model for individual economies, then introduce our new model. The details of our new model include FSV and an elaborated account of the sparsification. Section 4 illustrates some full-sample results by fitting our model to 34 economies. Section 5 is an out-of-sample forecasting exercise. We provide evidence that our proposed model can improve the forecasting overall and over time. Finally, Section 6 concludes.

2 Relationship to prior work

To make clear our contributions, we first briefly summarize the most closely related studies of the unobserved components model and global uncertainty. We then detail key differences in our analysis compared to the most closely related studies. In broad terms, our work extends the literature by a combination of allowing for cross-country linkages, the use of sparsification, and considering a large number of economies.

The unobserved components model A large body of research has emerged on extending the UC model. One strand of extensions has focused on introducing more indicators into the conditional mean. Another strand has focused on adding bounds on parameters. These extensions overlook the international comovement.

There has been a lot of recent research devoted to introducing suitable indicators into the UC model. These indicators are guided by either economic theory or empirical research. For instance, inspired by the Phillips curve, Stella and Stock (2013) extend the univariate UC model in Stock and Watson (2007) to a bivariate UC model, and assume that it is inflation gap and unemployment gap that drive the Phillips curve.[1] Based on public commentary that central bankers pay considerable attention to measures of long-run inflation expectations, Chan et al. (2018) develop a bivariate UC model by introducing survey-based long-run forecasts of inflation. To directly address critiques of omitted variable and omitted equation bias pointed out by Taylor and Wieland (2016), Zaman (2022) further extends the bivariate UC model to a large-scale UC model and jointly estimates trends of several macroeconomic variables. The observed flattening of Phillips curve has generated various explanations of this conundrum and some studies highlight the role played by global factors. Therefore, Kabundi et al. (2021) introduce global factors (global output and oil price) into the bivariate UC model. In this paper, we follow Stella and Stock (2013) to incorporate the Phillips curve into the UC model. One may question the existence of Phillips curve, but McLeay and Tenreyro (2020) emphasize that the Phillips curve exists and policymakers are completely aware of its existence. Hasenzagl et al. (2022) develop a model of inflation dynamics based on the view that the Phillips curve is one of the three important components. Stock and Watson (2008) raised the point that the Phillips curve is useful for conditional forecasting. So we expect that the Phillips curve still exists, even though we are observing that it has flattened (e.g., Ball and Mazumder, 2011; Hall et al., 2013 and Blanchard et al., 2015).

In parameter-rich models, it is common to use tight priors on coefficients or on error variances (or covariance matrices). And sometimes directly introducing restrictions on parameters can avoid them moving into undesirable regions. Such restrictions have been explored in many studies. For instance, Chan et al. (2013) bound both the inflation persistence to avoid the explosive region of the parameter

[1] Inflation gap is deviation of inflation from its trend, and similar interpretation of unemployment gap, deviation of unemployment rate from its trend.

space and the slope of Phillips curve to ensure a slope that is consistent with the economic theory. In this paper, we follow them to restrict the inflation persistence and the slope of Phillips curve. But there are some differences on the Phillips curve. We acknowledge that using output in log-levels gives the usual price Philips curve specification where the level of inflation is linked to the output gap as a measure of excess capacity. However, in this paper, we use output growth, not output in log-levels. The reasons are as follows. The trend-cycle decomposition might be sensitive to how the trend is modeled (see Perron and Wada, 2009; Grant and Chan, 2017). For the trend, log output is upward trending and only a drift is able to generate such a trend. Grant and Chan (2017) further find this drift for US is subject to structural breaks, but whether this is true for other countries is an empirical question. The focus of this paper is forecasting, not the estimate of output trend. For the output gap, it is a cycle and the econometric literature assumes that it is a stationary process with stochastic cyclical behavior. Prominent researchers have proposed various methods to impose the stationary condition on the AR(2) process. Which method would be suitable in a multi-country study is another empirical question.[2] For instance, it can be computationally-efficient to use the method by Grant and Chan (2017), where they directly bound the AR(2) coefficients. But Planas et al. (2008) stress that putting a prior on AR(2) coefficients is difficult to reproduce our knowledge and the implied distribution for the periodicity and amplitude can be counter-intuitive. Given the amplitude and periodicity of the cyclical movements, they propose to use trigonometric specification to re-parameterize the AR(2) process, but they exclude a moving-average term. Hasenzagl et al. (2022) brings back the moving-average term. If we use output growth, we do not need a stationary AR(2) process, thus avoiding to compare the various methods to impose a stationary condition. Output growth yet is not a measure of slack, but the use of growth as an alternative is not new. On the Taylor rule, Orphanides (2001) argues that a Taylor rule that reacts to output growth may be more stabilizing than a rule that responds to the output gap. Bullard and Eusepi (2005) develops a rule that responds to the growth gap, rather than output gap. On the Phillips curve, Sbordone (2002) derive a Phillips Curve as a function of trend growth. Mattesini and Nisticò (2010) study implications of trend growth on inflation dynamics. Tchatoka et al. (2017) compare the Phillips curve using output gap and growth. They find essentially results remain unchanged when employing output growth, so they concentrate on output growth. Gross and Semmler (2019) find the output growth correlates strongly with the slack measure (one-sided Hodrick-Prescott filter-based output gap) and could be an alternative to assess the empirical link between inflation and real activity.

As to the relationship of our paper to prior studies on the UC model, while our paper shares the two strands of extensions(introducing more indicators and constraining parameters to lie in reasonable intervals), we believe our paper provides further extensions. Firstly, we propose an approach to allow for cross-country linkages in the multi-country UC model. However, previous studies assume that countries are independent with each other. The idea that national macroeconomic developments depend on international conditions is not new. Kose et al. (2003) find that the world common component to expenditure time series of sixty countries explains between one-fourth and one-half of

[2] We thank the referee for pointing us in this direction.

the variance of these series in OECD countries. Ciccarelli and Mojon (2010) provide evidence that a simple average of 22 OECD countries' inflation accounts for almost 70% of the variance of inflation in these countries. So one aim of this paper is to study whether allowing for cross-country linkages will improve the forecast of variables in UC models. Secondly, we allow for cross-country linkages through global factors. These factors are estimated from the model. This is different from studies which use some specific variable to be a proxy of global factor. In the empirical application, we argue that introducing global factors will help reduce the omitted variable bias in a single-country UC model.

Global uncertainty Several ways to estimate global uncertainty have been proposed in the literature. Mumtaz and Theodoridis (2017) use a factor-augmented vector autoregression (VAR) model with a common stochastic volatility and a country-specific stochastic volatility. Pfarrhofer (2019) use a global vector autoregressive specification with FSV in the errors to estimate the impact of global uncertainty on six economies. Cuaresma et al. (2019) uses a large-scale Bayesian VAR with FSV to investigate the macroeconomic consequences of international uncertainty shocks in G7 countries. Carriero et al. (2020) measure international macroeconomic uncertainty by featuring the error volatility with a factor structure containing time-varying global components and idiosyncratic components.

As to the relationship of our paper to prior studies on measuring global uncertainty, our paper is closely related to the FSV specification used in Pfarrhofer (2019) and Cuaresma et al. (2019). The contribution of this paper is that we use the sparsification to avoid omitting some potentially important factors, whereas prior studies rely on expert judgement (either subjectively choose the number or rely on principal component-based analysis). The sparsification method, proposed by Chakraborty et al. (2020), obviates the need to specify a prior on the rank (in this paper, the rank is equivalent to the number of factors), and shrinks the regression matrix towards a low-rank structure. This sparsification method allows us to estimate the factors and use the post-processing to obtain an estimate for the number of factors.

Our FSV specification shares with Mumtaz and Theodoridis (2017) the feature of allowing for both common and country-specific stochastic volatility. It is empirically important to allow for stochastic volatility. Ignoring stochastic volatility is expected to exaggerate movements and potentially create transient variations in filtered estimates (see Sims, 2001; Stock, 2001; and Huber et al., 2020). One difference from Mumtaz and Theodoridis (2017) is that we use the sparsification to remove stochastic volatility in a data-based manner. This is important in the heavily parametrized setting. Similar strategy has been explored in Huber et al. (2020). In this paper, we shrink both factor volatilities and idiosyncratic volatilities. This is consistent with Carriero et al. (2020). They find, in the three-economy macroeconomic data set (USA, EA, and UK), the idiosyncratic component of volatility display very little time variation. Removing SV in a data-based manner is flexible since it can shrink small time-variation to zero while retains large time-variation (e.g. more volatile countries).

As regards the relationship of our paper to Carriero et al. (2020), there are mainly two differences. The first difference is about the number of factors. Carriero et al. (2020) rely on principal component-based analysis, while we use the sparsification and rely on post-processing. The second difference is the model in Carriero et al. (2020) features common factors in both volatilities and in the conditional mean of the VAR. In this paper, we only allow common factors to affect volatilities of the included variables. The reason that we do not allow common factors to affect the levels is that in the three-economy case, Carriero et al. (2020) find they will suffer from the convergence issue of the Markov chain Monte Carlo (MCMC) sampler if two common factors are both included in the conditional mean. So they include one common factor in the conditional mean. We might have the same issue since we include more factors and our data is shorter.

One other difference between our paper and a number of others in the multi-country studies is that we study 34 economies, including 23 advanced economies and 11 emerging market economies, whereas others focus on large economies, small advanced economies, or emerging market economies. As examples, Carriero et al. (2020) focus on large economies, Cross et al. (2018) focus on small advanced economies, Mumtaz and Theodoridis (2017) focus on eleven OECD countries, and Carrière-Swallow and Céspedes (2013) focus on emerging market economies.

3 Sparse Factor Stochastic Volatility for A Multi-country UC Model

This section begins by detailing the unobserved components model for individual economies, then introduces the factor stochastic volatility to allow for cross-country linkages. We refer to the model as a multi-country UC-FSV model. We then describe the sparsification. Finally, we summarize the model.

3.1 A Multi-country UC-FSV Model Specification

We begin with the UC model for output and inflation. In particular, we start from a constant coefficient UC model for inflation, $\pi_{i,t}$, and output growth, $y_{i,t}$ of the form:

$$\pi_{i,t} - \tau^{\pi}_{i,t} = \rho_i(\pi_{i,t-1} - \tau^{\pi}_{i,t-1}) + \alpha_i(y_{i,t} - \tau^{y}_{i,t}) + \varepsilon^{\pi}_{i,t}, \tag{1}$$

$$y_{i,t} - \tau^{y}_{i,t} = \varphi_{i,1}(y_{i,t-1} - \tau^{y}_{i,t-1}) + \varphi_{i,2}(y_{i,t-2} - \tau^{y}_{i,t-2}) + \varepsilon^{y}_{i,t}, \tag{2}$$

$$\tau^{\pi}_{i,t} = \tau^{\pi}_{i,t-1} + \varepsilon^{\tau\pi}_{i,t}, \quad \varepsilon^{\tau\pi}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{i\tau\pi}), \tag{3}$$

$$\tau^{y}_{i,t} = \tau^{y}_{i,t-1} + \varepsilon^{\tau y}_{i,t}, \quad \varepsilon^{\tau y}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{i\tau y}), \tag{4}$$

where i denotes economy i, $i = 1, \ldots, N$. At time t, $\pi_{i,t}$ is the inflation of economy i and $y_{i,t}$ is the output growth of economy i. $\tau^{\pi}_{i,t}$ and $\tau^{y}_{i,t}$ are their trends. These trends are unobserved latent states. In this paper, we refer to them as trend inflation and trend growth.

Equation (1) is inspired by the Phillips curve. We assume that it is inflation gap and growth gap that drive the Phillips curve. To ensure stationarity, we bound ρ_i and α_i to be positive and less than one, that is $0 < \rho_i < 1$ and $0 < \alpha_i < 1$, which also ensures that the Phillips curve has a positive slope. Chan et al. (2016) and Zaman (2022) also bound the coefficients and emphasize the importance of bounding.

Thus, the first equation embodies a Phillips curve, but we are assuming constant coefficients. Many papers have emphasized that the Phillips curve has flattened post 2007 (see, Simon et al., 2013) and proposed to allow for time-variation in the coefficients to capture this behavior (see, Zaman, 2022). It seems to be more sensible to start from a UC model with time-varying coefficients. However, using the data in our empirical work (from 1995Q1 to 2018Q1), we have considered a model where ρ_i and α_i vary over time, and found the Bayes Factor supports constant coefficients (see Appendix A). Accordingly, the main model does not have time-variation in the coefficients.

The second equation implies an AR(2) behavior for the growth gap. The AR(2) assumption is empirically sensible and commonly-used. Note that we are assuming constant coefficients in the growth gap equation. This assumption has also been used in Chan et al. (2016), Zaman (2022) and Kabundi et al. (2021). In the broader output literature, Koop et al. (2020) and Carriero et al. (2020) also assume constant coefficients.[3]

Equation (3) and (4) assumes a random walk process for trend inflation and trend growth. This specification is used in Cogley and Sbordone (2008). They find statistical models with time-varying drifts are able to explain quite well the behavior of inflation and output growth. For the time-varying drift, they assume it follows a random walk.

Thus far, we have specified a UC model for a single economy. In particular, it is a bivariate UC model, and incorporates the features from empirical findings (constant coefficients). However, conventional literature would next assume that the errors are independent across economies. It is with this assumption that we part with them.

As discussed earlier, the independent assumption across economies might not be plausible when there is significant commonality across economies. To capture such commonality in uncertainty, we assume that, for all economies, the errors in inflation gap equations are driven by common factors and the errors in growth gap equations are driven by common factors. This can be done through the factor stochastic volatility (FSV).

To facilitate the FSV specification, at time t, we store all errors in inflation gap equations in an N-dimensional vector $\boldsymbol{\varepsilon}_t^{\pi}$, that is, $\boldsymbol{\varepsilon}_t^{\pi} = (\varepsilon_{1,t}^{\pi}, \ldots, \varepsilon_{N,t}^{\pi})'$. $\varepsilon_{i,t}^{\pi}$ is the error for economy i. Similarly, we store all errors in growth gap equations in an N-dimensional vector $\boldsymbol{\varepsilon}_t^{y}$, that is, $\boldsymbol{\varepsilon}_t^{y} = (\varepsilon_{1,t}^{y}, \ldots, \varepsilon_{N,t}^{y})'$.

[3]Since we use output growth, we do not bound the constant coefficients in the growth gap equation.

$\varepsilon_{i,t}^{y}$ is the error for economy i. Through FSV, $\boldsymbol{\varepsilon}_t^{\pi}$ can be decomposed as:

$$\boldsymbol{\varepsilon}_t^{\pi} = L_{\pi}\boldsymbol{f}_t + \boldsymbol{u}_t^{\pi} \tag{5}$$

$$\begin{pmatrix} \boldsymbol{u}_t^{\pi} \\ \boldsymbol{f}_t \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0_N \\ 0_{r_\pi} \end{pmatrix}, \begin{pmatrix} \Sigma_t^{\pi} & 0_{r_\pi} \\ 0_N & \Omega_t^{\pi} \end{pmatrix} \right) \tag{6}$$

and $\boldsymbol{\varepsilon}_t^{y}$ can be decomposed as:

$$\boldsymbol{\varepsilon}_t^{y} = L_{y}\boldsymbol{g}_t + \boldsymbol{u}_t^{y} \tag{7}$$

$$\begin{pmatrix} \boldsymbol{u}_t^{y} \\ \boldsymbol{g}_t \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0_N \\ 0_{r_y} \end{pmatrix}, \begin{pmatrix} \Sigma_t^{y} & 0_{r_y} \\ 0_N & \Omega_t^{y} \end{pmatrix} \right) \tag{8}$$

where $\boldsymbol{f}_t = (f_{1,t}, \ldots, f_{r_\pi,t})'$ is a r_π-dimensional vector of latent factors and L_π is the associated $N \times r_\pi$ loading matrix. Similarly, $\boldsymbol{g}_t = (g_{1,t}, \ldots, g_{r_y,t})'$ is a r_y-dimensional vector of latent factors and L_y is the associated $N \times r_y$ loading matrix. Furthermore, we follow Chan (2022) to assume that the factor loading matrices L_π and L_y are both a lower triangular matrix with ones on the main diagonal and $r_\pi \leqslant (N-1)/2$, $r_y \leqslant (N-1)/2$.[4] Let $n_{l,\pi}$ denote the number of free elements in L_π, then $n_{l,\pi} = N \times r_\pi - \frac{(1+r_\pi)r_\pi}{2}$. Let $n_{l,y}$ denote the number of free elements in L_y, then $n_{l,y} = N \times r_y - \frac{(1+r_y)r_y}{2}$.

We assume that inflation gap equations across economies and growth gap equations across economies are driven by different factors $\boldsymbol{f}_t$ and $\boldsymbol{g}_t$. This assumption is for parsimony reasons. If the interest is in understanding the underlying causal relationship between output and inflation across countries, then the dependence assumption across the factors would make sense. One possible method is to assume both $\boldsymbol{f}_t$ and $\boldsymbol{g}_t$ follow a VAR process.[5]

Based on preliminary empirical work that errors in inflation gap equations exhibit stochastic volatility, we assume that the disturbances $\boldsymbol{u}_t^{\pi}$ exhibit stochastic volatility. This is why the error variance of $\boldsymbol{u}_t^{\pi}$ is Σ_t^{π}. Regarding growth gap equations, with the exception of Mertens (2014) and Zaman (2022), the previous literature would assume the errors remain homoscedastic, that is, $\boldsymbol{u}_t^{y}$ are homoscedastic. However, we assume the disturbances $\boldsymbol{u}_t^{y}$ exhibit stochastic volatility. This is why the error variance of $\boldsymbol{u}_t^{y}$ is Σ_t^{y}. Such specification will capture time variation in output variance unique to that economy. It has been used in Carriero et al. (2020) and Cesa-Bianchi et al. (2020). If the error is homoscedastic, our specification of the log-volatility can (nearly) remove SV in a data-based manner (through the Horseshoe prior).

[4] Chan et al. (2021) show that we do not need a lower triangular loading matrix when factors are heteroskedastic. If we stick to a lower triangular matrix, the prior will be order-dependent. In our empirical application section, we have experimented different orderings of economies. We find our conclusion does not change. So we simply follow the ordering from the database where we download the data. In Appendix G, we report some results about the number of factors when factor loading matrices are full.

[5] We leave this flavor to the future.

For the latent factors $\boldsymbol{f}_t$ and $\boldsymbol{g}_t$, we assume that they exhibit stochastic volatility. This is why the error variance of $\boldsymbol{f}_t$ is Ω_t^{π}, and the error variance of $\boldsymbol{g}_t$ is Ω_t^{y}.

The resulting time-varying variance matrix are $\Sigma_t^{\pi} = \text{diag}(e^{h_{1,t}^{\pi}}, \ldots, e^{h_{N,t}^{\pi}})$, $\Sigma_t^{y} = \text{diag}(e^{h_{1,t}^{y}}, \ldots, e^{h_{N,t}^{y}})$, $\Omega_t^{\pi} = \text{diag}(e^{h_{1,t}^{f}}, \ldots, e^{h_{r_\pi,t}^{f}})$, and $\Omega_t^{y} = \text{diag}(e^{h_{1,t}^{g}}, \ldots, e^{h_{r_y,t}^{g}})$.

We use $\exp(\boldsymbol{h}_t^{\pi}/2)$ to measure the **idiosyncratic inflation uncertainty**, $\exp(\boldsymbol{h}_t^{y}/2)$ to measure the **idiosyncratic growth uncertainty**, $\exp(\boldsymbol{h}_t^{f}/2)$ to measure the **global inflation uncertainty**, and $\exp(\boldsymbol{h}_t^{g}/2)$ to measure the **global growth uncertainty**. To facilitate the expression, we store the four types of log-volatilities in a N_h-dimensional vector $\boldsymbol{h}_t = (\boldsymbol{h}_t^{\pi}, \boldsymbol{h}_t^{y}, \boldsymbol{h}_t^{f}, \boldsymbol{h}_t^{g})$ where $N_h = 2N + r_\pi + r_y$. We summarize the definitions and descriptions of uncertainty in Table 1.

Table 1: Definitions and descriptions of uncertainty $\exp(\boldsymbol{h}_t/2)$

Definitions	descriptions of uncertainty $\exp(\boldsymbol{h}_t/2)$
idiosyncratic inflation uncertainty	$\exp(\boldsymbol{h}_t^{\pi}/2)$, the standard deviation of $\boldsymbol{u}_t^{\pi}$
idiosyncratic growth uncertainty	$\exp(\boldsymbol{h}_t^{y}/2)$, the standard deviation of $\boldsymbol{u}_t^{y}$
global inflation uncertainty	$\exp(\boldsymbol{h}_t^{f}/2)$, the standard deviation of $\boldsymbol{f}_t$
global growth uncertainty	$\exp(\boldsymbol{h}_t^{g}/2)$, the standard deviation of $\boldsymbol{g}_t$
global inflation factor	$\boldsymbol{f}_t$
global growth factor	$\boldsymbol{g}_t$

3.2 Sparsification

One of our contributions is the use of sparsification. We use the sparsification to estimate the factor loadings and rely on the post-processing to obtain an estimate for the number of factors. We also use the sparsification to remove stochastic volatility in a data-based manner. In this sub-section, we first talk about the number of factors, then removing stochastic volatility.

To facilitate the discussion, note that a generic Horseshoe prior takes the form

$$\beta_j \mid \lambda_j^{\beta}, \tau^{\beta} \sim \mathcal{N}\left(0, \lambda_j^{\beta}\tau^{\beta}\right), \tag{9}$$

$$\lambda_j^{\beta} \sim \mathcal{C}^{+}(0,1), \tag{10}$$

$$\tau^{\beta} \sim \mathcal{C}^{+}(0,1), \tag{11}$$

where $\mathcal{C}^{+}(\cdot,\cdot)$ denotes the half-Cauchy distribution, λ_j^{β} is the local shrinkage parameter and τ^{β} is the global shrinkage parameter. In Appendix H, we show the Horseshoe prior can take a hierarchical form using inverse-Gamma hyper-priors, and in the estimation we use the inverse-Gamma representation.

The number of factors This is done through the prior on the factor loading matrix. To avoid specifying a prior on the number of factors, Chakraborty et al. (2020) consider a potentially full-rank matrix and shrink out the redundant columns. Then they post-process the posterior draws to get the posterior estimate of the rank of the matrix (in this paper, the rank of a matrix is the number of factors). We follow their method. Theoretically, one can use a full matrix. In our case, this means setting the number of factors to the number of economies N. However, we do not do this.[6] Thanks to the bulk of empirical studies, some guidance of the number of factors is available. We choose a slightly higher number.[7]

To assign shrinkage priors on factor loading matrices (L_π and L_y), we use the Horseshoe prior and specify a column-specific global shrinkage parameter and an element-specific local parameter. For instance, for L_π, let $L_{\pi,j}$ denote the j-th column of factor loading matrix L_π, then the prior on the i-th element in $L_{\pi,j}$ is the Horseshoe prior with a column-specific global shrinkage parameter $(L_{\pi,j})$ and an element-specific local parameter $(L_{\pi,ij})$.

The final step is to post-process the posterior draws. We threshold the singular values of the factor loading matrix, and estimate the rank as the number of nonzero thresholded singular values. We refer our readers to Chakraborty et al. (2020) for more details.

To remove stochastic volatility To allow the data to decide whether there is time-variation in their log-volatility, we model the evolution of the log-volatility as a random walk. This random walk is in non-centered parameterization. Then we use a global-local shrinkage prior (the Horseshoe prior) to control time-variation. More specifically, for each $j = 1, \ldots, N_h$, the evolution of the log-volatility is modeled as:

$$\begin{aligned} h_{j,t} &= h_{j,0} + \omega_j^h \widetilde{h}_{j,t} \\ \widetilde{h}_{j,t} &= \widetilde{h}_{j,t-1} + +\varepsilon_{j,t}^h, \quad \varepsilon_{j,t}^h \sim \mathcal{N}(0,\ 1) \end{aligned} \tag{12}$$

The non-centered parameterization decomposes a time-varying parameter $h_{j,t}$ into two parts: a time-invariant part $h_{j,0}$ and a time-varying part $\omega_j^h \widetilde{h}_{j,t}$. The time-varying part has a constant coefficient ω_j^h, which controls the time-variation. If the error is homoscedastic, then we expect ω_j^h may be (or close to) zero. If the error is heteroscedastic, then we expect ω_j^h is different from zero. This case is exactly the advantage of global-local shrinkage priors. Many papers have documented that global-local shrinkage priors can cope with the case where a matrix is characterized by zero and non-zero elements (e.g., Polson and Scott, 2010; Kastner and Huber, 2020). So we use the empirically successful

[6]In fact we have experimented this. But we find the computation becomes a burden and find the forecast performance does not improve much.

[7]For instance, Carriero et al. (2020) find there is one global factor driving the 19-country GDP. What we did is to set the number of global output factors to two.

global-local shrinkage prior, Horseshoe prior for ω_j^h.

If the error (factor) really is homoscedastic, the Horseshoe prior will shrink ω_j^h to (nearly) zero and automatically remove (or nearly so) the SV from the error (factor). The Horseshoe prior has a global shrinkage parameter. It will push all elements (ω_j^h) towards zero. We assume that there exists a single global shrinkage parameter. This is a restricted version of the Horseshoe prior in Feldkircher et al. (2021). They specify the global shrinkage parameter to differ across economies and equations within a given economy. However, we notice that such a flexible prior is used for the coefficients in their panel VARs. Our Horseshoe prior is for the time-varying part of log-volatility. Since the log-volatilities all represent the uncertainty, we expect that they have a single global shrinkage parameter. To capture the differences across factors, economies and equations, we rely on the local shrinkage parameter.

For ω_j^h, we consider the Horseshoe prior. For the time-invariant part of log-volatility, $h_{j,0}$, we also consider the Horseshoe prior. Such priors might be too strong on log-volatility, so in Appendix G, we consider a normal prior with zero mean and variance one on $h_{j,0}$.

To complete the priors, we assume that the constant coefficients and initial states (ρ_i, α_i, $\varphi_{i,j}$, $\tau_{i,1}^{\pi}$, $\tau_{i,1}^{y}$) follow normal distribution with zero mean and variance ten.[8] The error variances ($\sigma_{\tau\pi}^2$ and $\sigma_{\tau y}^2$) are assumed to follow inverse gamma distribution $\mathcal{IG}(10,\ 0.18)$.

3.3 Summarizing the model

To summarize the model including all economies:

$$\begin{aligned}
\boldsymbol{\pi}_t - \boldsymbol{\tau}_t^{\pi} &= P(\boldsymbol{\pi}_{t-1} - \boldsymbol{\tau}_{t-1}^{\pi}) + A(\boldsymbol{y}_t - \boldsymbol{\tau}_t^{y}) + L_{\pi}\boldsymbol{f}_t + \boldsymbol{u}_t^{\pi},\ \boldsymbol{f}_t \sim \mathcal{N}(0,\ \Omega_t^{\pi}),\ \boldsymbol{u}_t^{\pi} \sim \mathcal{N}(0,\ \Sigma_t^{\pi}) \\
\boldsymbol{y}_t - \boldsymbol{\tau}_t^{y} &= \Phi_1(\boldsymbol{y}_{t-1} - \boldsymbol{\tau}_{t-1}^{y}) + \Phi_2(\boldsymbol{y}_{t-2} - \boldsymbol{\tau}_{t-2}^{y}) + L_y\boldsymbol{g}_t + \boldsymbol{u}_t^{y},\ \boldsymbol{g}_t \sim \mathcal{N}(0,\ \Omega_t^{y}),\ \boldsymbol{u}_t^{y} \sim \mathcal{N}(0,\ \Sigma_t^{y}) \\
\tau_{i,t}^{\pi} &= \tau_{i,t-1}^{\pi} + \varepsilon_{i,t}^{\tau\pi},\quad \varepsilon_{i,t}^{\tau\pi} \sim \mathcal{N}(0,\ \sigma_{i\tau\pi}^2),\ i = 1, \ldots, N \\
\tau_{i,t}^{y} &= \tau_{i,t-1}^{y} + \varepsilon_{i,t}^{\tau y},\quad \varepsilon_{i,t}^{\tau y} \sim \mathcal{N}(0,\ \sigma_{i\tau y}^2) \\
h_{j,t} &= h_{j,0} + \omega_j^h \widetilde{h}_{j,t} \\
\widetilde{h}_{j,t} &= \widetilde{h}_{j,t-1} + \varepsilon_{j,t}^{h},\quad \varepsilon_{j,t}^{h} \sim \mathcal{N}(0,\ 1),\ j = 1, \ldots, N_h
\end{aligned} \tag{13}$$

where $\boldsymbol{\pi}_t = (\pi_{1,t}, \ldots, \pi_{N,t})'$ is an $N \times 1$ vector, $\boldsymbol{\tau}_t^{\pi} = (\tau_{1,t}^{\pi}, \ldots, \tau_{N,t}^{\pi})'$ is an $N \times 1$ vector, $P = \text{diag}(\rho_1, \ldots, \rho_N)$ is an $N \times N$ matrix, $A = \text{diag}(\alpha_1, \ldots, \alpha_N)$ is an $N \times N$ matrix, $\boldsymbol{y}_t = (y_{1,t}, \ldots, y_{N,t})'$ is an $N \times 1$ vector, $\boldsymbol{\tau}_t^{y} = (\tau_{1,t}^{y}, \ldots, \tau_{N,t}^{y})'$ is an $N \times 1$ vector, $\Phi_1 = \text{diag}(\phi_{1,1}, \ldots, \phi_{N,1})$ is an $N \times N$ matrix, $\Phi_2 = \text{diag}(\phi_{1,2}, \ldots, \phi_{N,2})$ is an $N \times N$ matrix.

We will use the multi-country UC-FSV as an acronym for this model defined through equation (13). Many models can be written as a restricted version of the multi-country UC-FSV model. These restrictions can help to investigate some aspects of our model. The restricted models, along with

[8]Remember that we bound ρ_i and α_i to be positive and less than one.

their acronyms, are as follows:

1) UC-FSV-$r_y = 0$: this is the restricted version of the UC-FSV where there is no common factors in growth gap equations, that is, $r_y = 0$. And errors in growth gap equations are allowed to exhibit stochastic volatility.

2) UC-FSV-$r_y, r_\pi = 0$: this is the restricted version of UC-FSV where there is no common factors in inflation gap and growth gap equations, that is, $r_\pi = 0$, $r_y = 0$. Errors in inflation gap and growth gap equations are allowed to exhibit stochastic volatility.

3) UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$: this is the restricted version of UC-FSV where there is no common factors in inflation gap and growth gap equations, that is, $r_\pi = 0$, $r_y = 0$, and errors in growth gap equations u_t^y are homoscedastic, while errors in inflation gap equations u_t^π exhibit stochastic volatility. This is the model that is used in Stella and Stock (2013), and Chan et al. (2016).[9]

4 Full-sample results

4.1 Data

The data are the quarterly consumer price index (CPI) and the quarterly real gross domestic product (GDP) for 34 economies, 23 advanced economies (AEs)[10] and 11 emerging market economies (EMEs).[11] They span the period from 1995Q1 to 2018Q1. The choice of countries and the sample size is based on data availability. The series included are the headline consumer price index (CPI) representing domestic headline inflation and real gross domestic product (GDP) which reflects domestic demand. Real GDP data are obtained from Haver Analytics. We transform the data to annualized growth rates as: $400\log(z_t/z_{t-1})$. And because the growth gap equation follows an AR(2) process, our estimation start from 1995Q4. We set $r_\pi = 5$ and $r_y = 2$, that is, we include five factors in inflation gap equations and two factors in growth gap equations. Posterior results are based on 100,000 draws after a burn-in period of 20,000.

4.2 Overview of empirical results

We divide our full-sample results into three sub-sections. The first sub-section is the estimate of global inflation uncertainty $\exp(\boldsymbol{h}_t^f/2)$ and global growth uncertainty $\exp(\boldsymbol{h}_t^g/2)$. Then we report the correlation between global inflation factors and domestic business cycle.

[9] The coefficients in this paper are restricted to be constant.

[10] Australia, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Israel, Italy, Latvia, Lithuania, Netherlands, Portugal, Slovakia, South Korea, Spain, Sweden, Switzerland, UK, USA.

[11] Bolivia, Brazil, China, Hungary, Indonesia, Mexico, Philippines, Russia, South Africa, Thailand, Turkey.

The second sub-section is the sparsification. We use the sparsification to avoid omitting some potentially important factors and remove stochastic volatility in a data-based manner. We report the posterior estimate of the number of factors. The evidence of removing stochastic volatility are provided in Appendix F.

The third sub-section is the Bayesian model comparison. We compare the multi-country UC-FSV to alternative models (UC-FSV-$r_y = 0$, UC-FSV-$r_y, r_\pi = 0$, UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) described in Section 3.3.

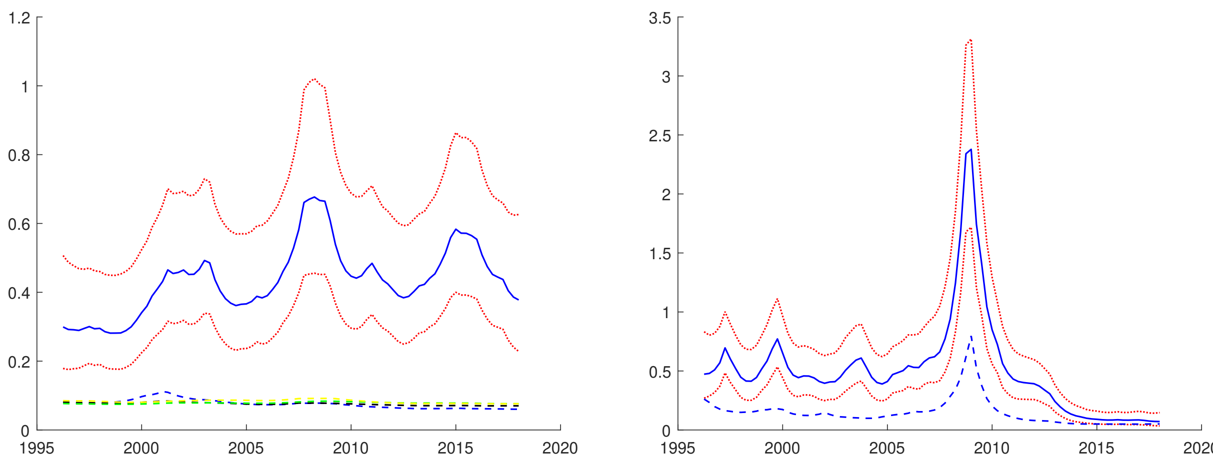
4.3 Estimates of global uncertainty

Although the multi-country UC-FSV estimates of global uncertainty reflect contemporaneous effect of global factors on (the volatility of) macroeconomic data, the effect is also directly related to the loadings on global factors. These loadings are reported in Appendix B. We report the posterior mean of the five factors' loadings (recall that we set $r_\pi = 5$), but only the 16% and 84% quantiles of first factor's loadings for brevity. Most of the economies have sizable loadings on the first global inflation factor, and the quantiles (except Russia and Brazil) do not include zero. Then we report the loadings on global growth factor. The quantiles of the first global growth factor for all economies do not include zero. This provides strong evidence of significant commonality of output growth in the 34 economies. Carriero et al. (2020) obtain similar result in their case of the 19-country GDP dataset.

Figure 1 displays the posterior estimates of global uncertainty obtained from the multi-country UC-FSV using the full sample. The left panel is the estimate of global inflation uncertainty, and the right panel is global growth uncertainty. In both figures, the solid lines represent the posterior means of the first uncertainty, while the dotted lines are the associated 16% and 84% quantiles. The dashed lines represent the posterior means of the remaining uncertainties. For instance, with regard to global inflation uncertainty, we set $r_\pi = 5$, so we obtain the posterior estimates of the five global inflation uncertainties from MCMC, including their posterior means and quantiles. Then, in Figure 1 (a), we plot the posterior means and quantiles of the first global inflation uncertainty (see solid lines and dotted lines), but for brevity, we only plot the posterior means of the remaining uncertainties (the second, third, fourth and fifth uncertainty) using dashed lines.

As indicated in Figure 1 (a), we only observe evident and meaningful time-variation in the first global inflation uncertainty. The estimated global inflation uncertainty show significant increases around some of the political and economic events that Bloom (2009) highlights as periods of uncertainty, including 9/11, the Enron scandal, the second Gulf war, and the global financial crisis period. These spikes associated with the global factor are documented in Kastner and Huber (2020) using US macroeconomic data. Since our data comes from 34 economies, the consistency (between the estimate in Kastner and Huber (2020) and our study) indicates that global macroeconomic uncertainty is closely related to uncertainty in the US, which might not seem surprising given the tie of the international economy to the US economy. One spike that is not documented in Kastner and Huber (2020) is that volatility increases from 2013 onward. This may indicate that such increase is driven by economies other than US. In addition, at the end of our sample (2018Q1), the global inflation uncertainty still exists and continues to influence all economies under consideration. This is supported by a related study, Forbes (2019). They add commodity price volatility to explain inflation and find that commodity price volatility plays a large role for CPI inflation.

However, we find a different story with regard to the time-variation in the global growth uncertainty from Figure 1 (b). First, the two global growth uncertainties both increase during the GFC of 2008, but except this, we do not observe other meaningful time-variation form the second global growth uncertainty.[12] Before the Global Financial Crisis (GFC) of 2008, there exists global growth uncertainty but it does not show much time-variation. Then during the GFC, such uncertainty increases substantially. In the aftermath of the GFC, it decreases sharply and importantly, in the 2015, the global inflation uncertainty reaches a very low level.[13] These features are documented in Carriero et al. (2020) in their 19-country GDP data set.



(a) Global inflation uncertainty

(b) global growth uncertainty

Figure 1: Posterior estimates for global inflation uncertainty $\exp(\boldsymbol{h}_t^f/2)$ and global growth uncertainty $\exp(\boldsymbol{h}_t^g/2)$ under the multi-country UC-FSV. The solid lines represent the posterior means of the first global uncertainty, while the dotted lines are the associated 16% and 84% percentiles. The dashed lines represent the posterior means of the remaining uncertainties.

Next Table 2 reports the correlation between global inflation factors $\boldsymbol{f}_t$ and domestic business cycle $(y_{i,t} - \tau_{i,t}^y)$ on average. We check this correlation mainly because we get a lower α when adding FSV (See Table 13 in Appendix C). Recall that α is the slope of the Phillips curve. This may give us an impression that allowing for cross-country linkages will flatten the Phillips curve. But we think we need more care to interpret this lower value. So we check the correlation between two variables:

[12]This is the first reason of including only two factors in the growth gap equation.
[13]This is the second reason of including only two factors in the growth gap equation.

global inflation factors $f_{1,t}$[14] and domestic business cycle $(y_{i,t} - \tau^{y}_{i,t})$. Then we take average across economies.

The results show that the estimated global inflation factor is positively correlated with domestic business cycle. We interpret this as part of the variation captured in the global inflation factor reflects reflects a global business cycle. Introducing factors could reduce the omitted variable bias.

Table 2: Posterior estimates of correlation between global inflation factors and domestic business cycle

	Correlation
Mean	0.22
16% quantile	0.19
84% quantile	0.25

4.4 Sparsification: The number of factors

To obtain the posterior estimate of the number of factors, we post-process the posterior draws as: threshold the singular values of the factor loading matrix, and estimate the rank as the number of nonzero thresholded singular values. One choice of the threshold is proposed in Chakraborty et al. (2020). Using their choice, we get the result in Table 3. We have inflation gap equations and growth gap equations. The second column is singular values for inflation gap equations, and the third column is singular values for growth gap equations. The first row reports the threshold. The number of factors is determined as: the number of singular values larger than the threshold. In growth gap equations, we find that there is one singular value (119.82) that is larger than the threshold (= 108.46). This means that there is one global factor in growth gap equations. This finding is consistent to prior studies.

In inflation gap equations, we find that there is no singular value that is larger than the threshold (= 118.47). This means that there is no global factor in the inflation equation.[15] Even if in the full-sample result we do not find strong evidence of global factors in inflation gap equations, in the out-of-sample forecasting exercise we find the FSV specification does improve the forecast of inflation (although the improvement of forecasting inflation is smaller than the improvement of forecasting

[14] $f_{1,t}$ is the global inflation factor with highest variation. Since other factors are quite flat and do not have meaningful interpretation, we do not consider them.

[15] In Appendix G, we provide additional results that provide robustness checks on the estimate of the number of factors. Two dimensions to which I assess the robustness are: (1) The identification constraint on the factor loading matrices L_π and L_y; and (2) the shrinkage on the time-invariant part of log-volatility $h_{j,0}$. Our conclusion remain the same. Another method about the number of factors is to choose the number of factors by the model's forecast performance. For instance, a table reporting the sum of one-step-ahead log predictive likelihoods in Table 4 but with different numbers of factors.

growth. See Section 5).

Table 3: Posterior number of factors

Singular values (Descending)	The inflation equation threshold = 118.47	The output equation threshold = 108.46
First	49.08	119.82
Second	23.33	45.85
Third	5.17	20.62

4.5 Bayesian Model Comparison

As discussed previously, the computation of marginal likelihood can be a challenge when there are a large number of states. Therefore, we use an approximation to the marginal likelihood (see Geweke, 2001; Geweke and Amisano, 2010; and Cross et al., 2020). They propose that conditioning on the estimation period, the sums of one-step-ahead joint log predictive likelihoods of 34 economies can be viewed as an approximation to the marginal likelihood, therefore provides a direct measure of in-sample fit. We compare four competing models: the multi-country UC-FSV, UC-FSV-$r_y = 0$, UC-FSV-$r_y, r_\pi = 0$ and UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

Before computing the the sums of one-step-ahead joint log predictive likelihoods, we need to define some basics. Let $\widehat{y}_{t+k}^{(i,j)}$ denote, at time t, the k-step-ahead forecast of the j-th variable in the i-th economy, and $y_{t+k}^{(i,j)}$ denote the actual value. In our empirical work, $i = 1, \ldots, N$ with $N = 34$, $j = 1, 2$ where $j = 1$ denote inflation and $j = 2$ denote growth. $\mathrm{Y}_{1:t}^{(i,j)}$ stores the data up to time t, so $\widehat{y}_{t+k}^{(i,j)} = \mathrm{E}\ (y_{t+k}^{(i,j)} \mid \mathrm{Y}_{1:t}^{(i,j)})$. Then we compute the k-step-ahead log predictive likelihoods (LPL) at time t of the i-th economy the j-th variable:

$$\mathrm{LPL}_{t,i,j,k} = \log\, p(\widehat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \mathrm{Y}_{1:t}^{(i,j)}),\ t = T_0, \ldots, T-k$$

Then the sums of one-step-ahead joint log predictive likelihoods is computed using:

$$\mathrm{LPL}_{\cdot,\cdot,\cdot,1} = \sum_{t=T_0}^{T-1} \sum_{i=1}^{n} \sum_{j=1}^{2} \log\, p(\widehat{y}_{t+1}^{(i,j)} = y_{t+1}^{(i,j)} | \mathrm{Y}_{1:t}^{(i,j)})$$

Our estimation period starts from 1995Q4 (to 2018Q1), and the forecasting evaluation period starts from 2003Q1. We provide the sums of one-step-ahead joint log predictive likelihoods of 34 economies in Table 4.

In Table 4, results are presented relative to the forecast performance of the UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$: we take differences, so that a positive number indicates a model is forecasting better than the UC-

FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.[16] The results show that the multi-country UC-FSV provides the best fit compared to all other models. In addition, since we find UC-FSV-$r_y, r_\pi = 0$ provides higher model fit than UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$, we view this as another evidence in support of allowing for idiosyncratic stochastic volatility in growth gap equations.

Table 4: Sum of one-step-ahead log predictive likelihood

Model	against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$
UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$	0
UC-FSV-$r_y, r_\pi = 0$	520.37
UC-FSV-$r_y = 0$	658.57
UC-FSV	883.34

5 Out-of-sample Forecasting Results

Since our modifications are about uncertainty, we focus on the density forecast. We use the data from 1995Q4 to 2002Q4 as an initial estimation period, and use data through 2002Q4 to produce k-step-ahead forecast distributions. We consider forecast horizons of $k = 1, 4, 8, 12, 16$ quarters. So our forecast evaluation period begins in 2003Q1. We divide our out-of-sample forecasting results into three parts: forecasting inflation, forecasting output growth and jointly forecasting inflation and output growth. For each part, we discuss the results in three dimensions. The first dimension is aggregate forecasting performance over time and across economies (the aggregate LPL, by summing all economies and all time periods). Since we observe international macroeconomic uncertainty, it is natural to expect that considering such uncertainty will provide more accurate forecast in economic recession. Thus, the second dimension is about forecasting performance over time (we can study how the sums of LPL changes over time, by summing all economies at time t). After providing evidence that our multi-country UC-FSV can produce more accurate forecast in economic recession, we further study whether such good forecast performance is driven by particular economies, so the third dimension is about the forecasting performance at economy level. All results are presented relative to the forecast under UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$: we take differences, so a positive number indicates a model is forecasting better than the UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

5.1 Forecasting inflation

We first report the aggregate forecasting performance for inflation over time and over economies in Table 5. It is calculated by summing the LPL for the N economies over T_0 to $T - k$ (and recall that

[16] Please note that we only take the sum, and no average. That may be why the number seems so large. For instance, the sums of LPL under UC-FSV is 895.02. If we take average over time, then it is 14.67. If we take further average across economies, then it is 0.43.

$j = 1$ denote inflation):

$$\text{LPL}_{\cdot,\cdot,1,k} = \sum_{t=T_0}^{t=T-k} \sum_{i=1}^{n} \log p(\widehat{y}_{t+k}^{(i,1)} = y_{t+k}^{(i,1)} | \text{Y}_{1:t}^{(i,1)})$$

The results show that the model with cross-country linkages in inflation (UC-FSV-$r_y = 0$ and UC-FSV) provides more accurate forecast for inflation than the model without cross-country linkages (UC-FSV-$r_y, r_\pi = 0$ and UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) at all horizons.

Table 5: Sum of k-step-ahead log predictive likelihood for 34-country inflation

Model	k=1	k=4	k=8	k=12	k=16
UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$	0	0	0	0	0
UC-FSV-$r_y, r_\pi = 0$	-4.27	71.83	127.82	138.85	185.26
UC-FSV-$r_y = 0$	98.92	**265.09**	286.02	350.53	333.11
UC-FSV	**101.63**	257.39	**294.76**	**379.19**	**356.89**

The forecasting result of inflation in Table 5 suggests the benefits of allowing for cross-country linkages, which is done through considering the global inflation uncertainty in our paper. It is natural to expect that the good forecasting result may largely arise from periods of uncertainty. To investigate this point, we calculate the sums of LPL over time. A common method is, as done in Feldkircher et al. (2021), to sum the LPL for the N economies at time t:

$$\text{LPL}_{t,\cdot,1,k} = \sum_{i=1}^{n} \log p(\widehat{y}_{t+k}^{(i,1)} = y_{t+k}^{(i,1)} | \text{Y}_{1:t}^{(i,1)})$$

For instance, suppose we are at the time point of 2007Q4, then $k = 1$ means we are forecasting the data in 2008Q1, and $k = 4$ means we are forecasting the data in 2008Q4. So this method helps to answer at time t, which model can provide the most accurate forecast in the future.

However, recall that global inflation uncertainty shows significant increases around 2008 and 2015 (see Figure 1 (a), and because our forecast starts from 2003Q1, so we omit the increase in 2001). Such global inflation uncertainty drives strong co-movement across economies. So a more interesting study is to investigate whether this global inflation uncertainty can improve the forecast performance during periods of uncertainty. For instance, suppose that we want to know which model can provide the most accurate forecast of 2008Q1? Different forecast horizons will provide the forecast made at different time t. If $k = 1$, then this means the forecast is made at 2007Q4 (one-step-ago). If $k = 4$, then this means the forecast is made at 2007Q1 (four-step-ago). Overall, the difference is the X axis. Suppose that we are at time t, in Feldkircher et al. (2021), the X axis is t and represents when we make the forecast, but in our paper, the X axis is $t + k$ and represents when to forecast. That is how we produce Figure 2. About the starting time, since we make the first forecast at 2002Q4, if $k = 1$, the time to forecast (at 2002Q4) is 2003Q1, so in Figure 2, the X axis (time to forecast) starts from 2003Q1 when $k = 1$. If $k = 4$, the time to forecast (at 2002Q4) is 2003Q4, so in Figure 2, the X axis (time to forecast) starts from 2003Q4 when $k = 4$. Similarly, if $k = 16$, the time to forecast (at 2002Q4) is 2006Q4, so in Figure 2, the X axis (time to forecast) starts from 2006Q4 when $k = 16$.

We plot the results (against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Figure 2 (for brevity, we only plot the results of UC-FSV). To forecast inflation during periods of uncertainty (like 2008), we find overall good forecast performance for UC-FSV at all horizons, particularly at long horizons. This indicates the importance of taking into account cross-country linkages for improving forecasts of inflation, especially to forecast periods of uncertainty. To forecast more stable periods, it does not harm to take into account cross-country linkages.

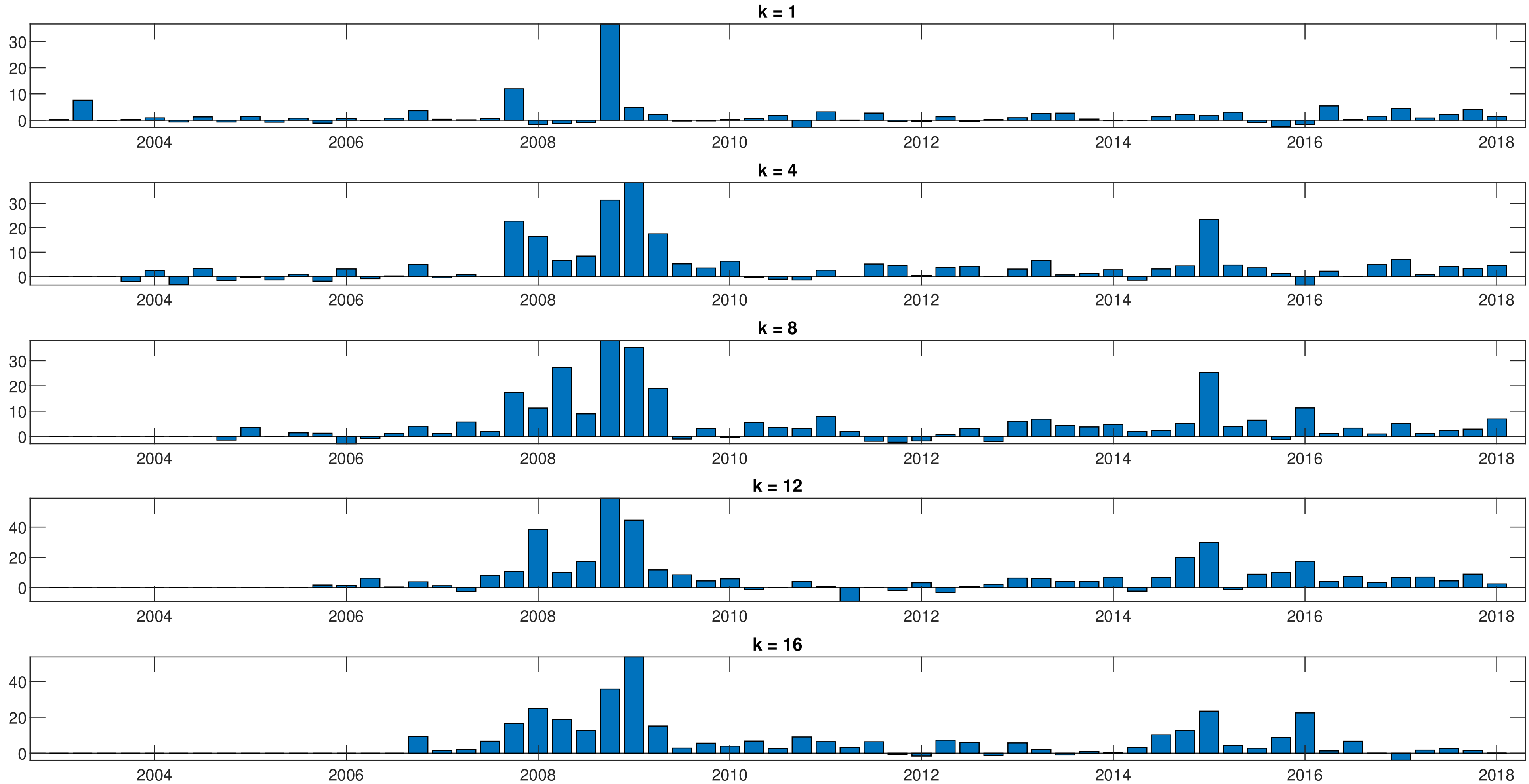


Figure 2: Sums of k-step ahead LPL of inflation for UC-FSV relative to UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$ over time. The X axis is $t + k$ and represents when to forecast.

The sums of LPL over time in Figure 2 is for the 34 economies. Someone may question whether the good forecasting result is driven by particular economies? To investigate this point, we present the forecasting result for individual economies. The LPL of inflation for economy i at time t, which can be calculated by:

$$\mathrm{LPL}_{t,i,1,k} = \log p(\widehat{y}_{t+k}^{(i,1)} = y_{t+k}^{(i,1)} | \mathrm{Y}_{1:t}^{(i,1)})$$

We plot the results (against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Figure 3. Here the period of uncertainty that we plot is 2008Q4, so time to forecast is 2008Q4 ($t + k = 2008Q4$). If $k = 1$, then the time we make forecast is 2008Q3, and we find overall good forecast performance for most economies with more pronounced gains in advanced economies (The first 23 economies are AEs, and the following 11 economies are EMEs). A similar pattern is found if $k = 16$. The time we make forecast is 2004Q4, and we also find overall good forecast performance for most economies. We also find significant gains in Spain and USA. The gain is not so significant if $k = 1$ as the gain if $k = 16$. In Figure 3, we only plot the shortest horizon $k = 1$ and the longest horizon $k = 16$, for middle horizons ($k = 4, 8, 12$), we find good forecasting result across most economies and did not find a particular economy which is important in driving good forecasting results. Overall, We find good forecast performance for UC-FSV for most economies and such good forecast performance is not driven by particular economies.

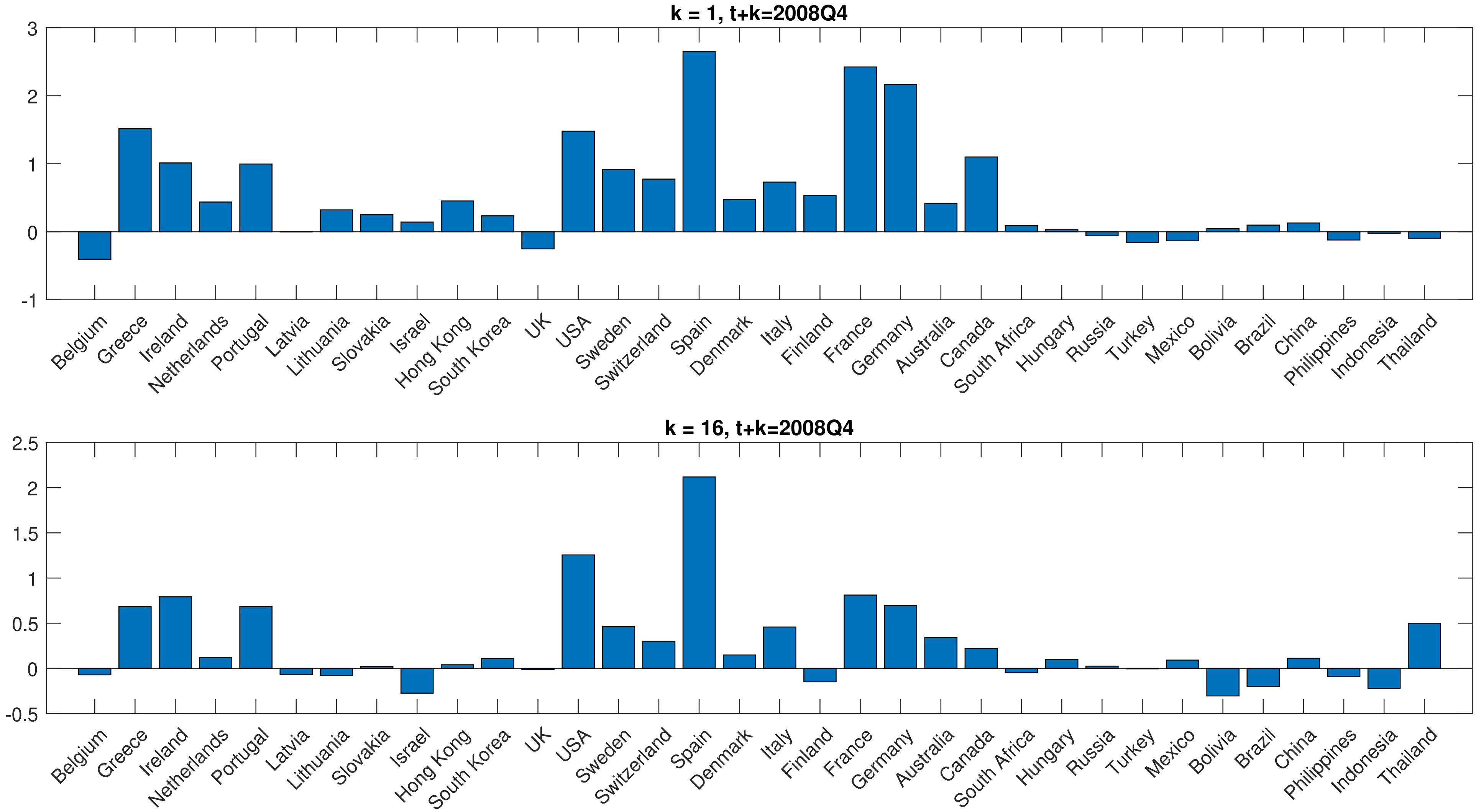


Figure 3: Sums of k-step ahead LPL of inflation for economy i under UC-FSV relative to UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

5.2 Forecasting output growth

With regard to output growth, we report the sums of LPL of output over time and over economies in Table 6. It is calculated by summing the LPL for the N economies over T_0 to $T-k$ (and recall that $j=2$ denote output growth):

$$\mathrm{LPL}_{\cdot,\cdot,2,k} = \sum_{t=T_0}^{t=T-k} \sum_{i=1}^{n} \log p(\widehat{y}_{t+k}^{(i,2)} = y_{t+k}^{(i,2)} | \mathrm{Y}_{1:t}^{(i,2)})$$

The results show that the model, which allows for both idiosyncratic stochastic volatility and cross-country linkages in growth gaps, provides the most accurate forecast for output growth at all horizons.

Table 6: Sum of k-step-ahead log predictive likelihood for 34-economy output growth

Model	k=1	k=4	k=8	k=12	k=16
UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$	0	0	0	0	0
UC-FSV-$r_y, r_\pi = 0$	577.02	694.98	811.01	797.25	684.98
UC-FSV-$r_y = 0$	566.81	668.04	852.04	772.90	680.99
UC-FSV	**762.93**	**1194.99**	**1211.17**	**1208.10**	**1052.36**

Similar to the analysis of inflation, the second dimension of discussion for output growth is sums of LPL over time (by summing all economies at time t), which can be calculated by:

$$\mathrm{LPL}_{t,\cdot,2,k} = \sum_{i=1}^{n} \log p(\widehat{y}_{t+k}^{(i,2)} = y_{t+k}^{(i,2)} | \mathrm{Y}_{1:t}^{(i,2)})$$

We plot the results (against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Figure 4. To forecast output growth during periods of uncertainty (like 2008), we find overall good forecast performance for UC-FSV at all horizons. This indicates the importance of taking into account cross-country linkages for improving forecasts of output growth, especially to forecast periods of uncertainty. To forecast more stable periods, it does not harm to take into account cross-country linkages.

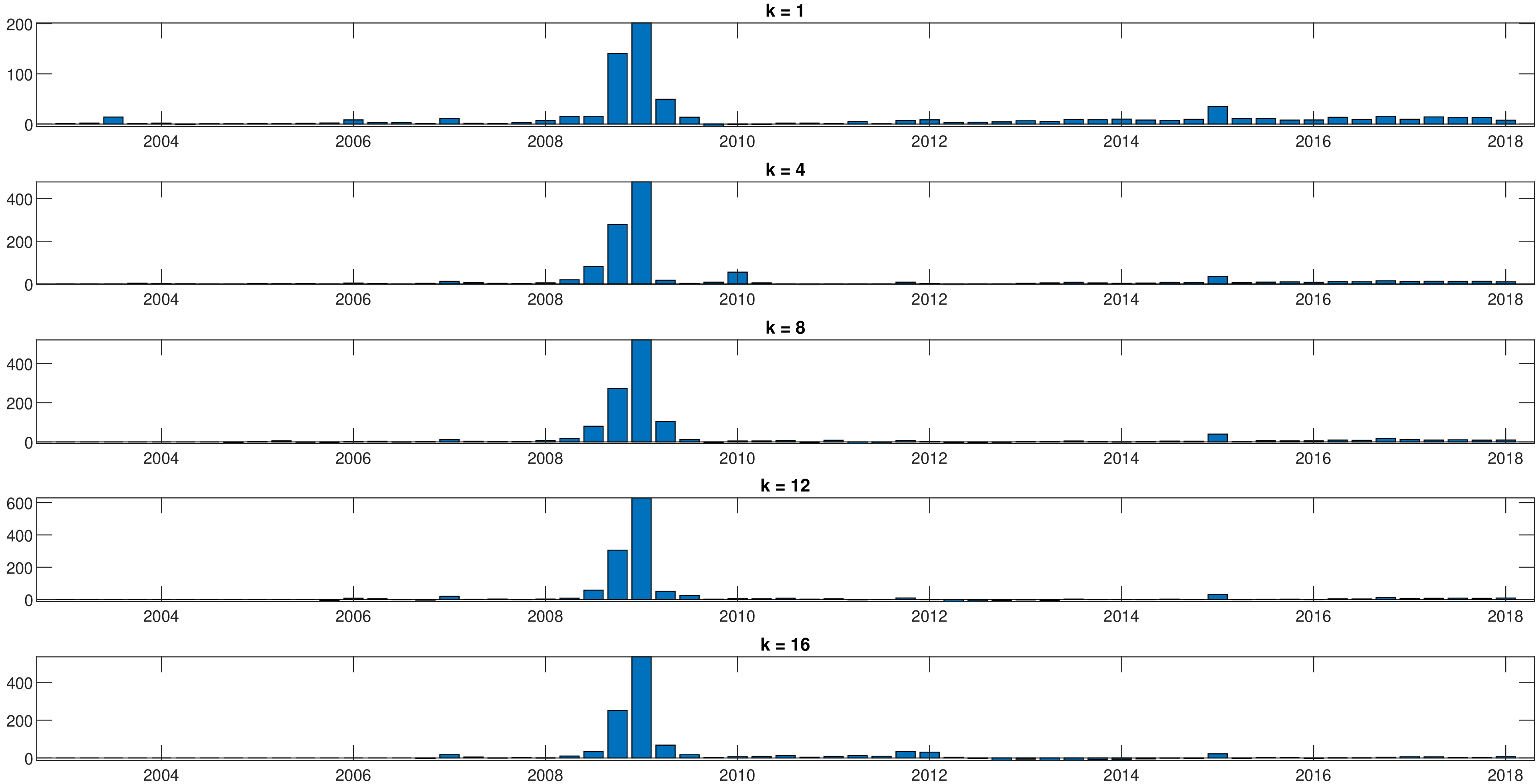


Figure 4: Sums of k-step ahead LPL of output for UC-FSV relative to UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$ over time. The X axis is $t + k$ and represents when to forecast.

To investigate whether the good forecast performance is driven by particular economies, we calculate the sums of LPL of output growth for economy i at time t by:

$$\mathrm{LPL}_{t,i,2,k} = \log\, p(\widehat{y}^{(i,2)}_{t+k} = y^{(i,2)}_{t+k} | \mathrm{Y}^{(i,2)}_{1:t})$$

We plot the results (against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Figure 5. We choose 2008Q4 to represent the period of uncertainty. For $k = 1$ and $k = 16$, we both find overall good forecast performance for UC-FSV for all economies. The highest gain is found for Hungary, followed by Sweden. However, different from the conclusion in the case of forecasting inflation that more pronounced gains are found in AEs, we find significant gains in both AEs and EMEs. This implies that allowing for idiosyncratic stochastic volatility and cross-country linkages in output growth is important for both AEs and EMEs.

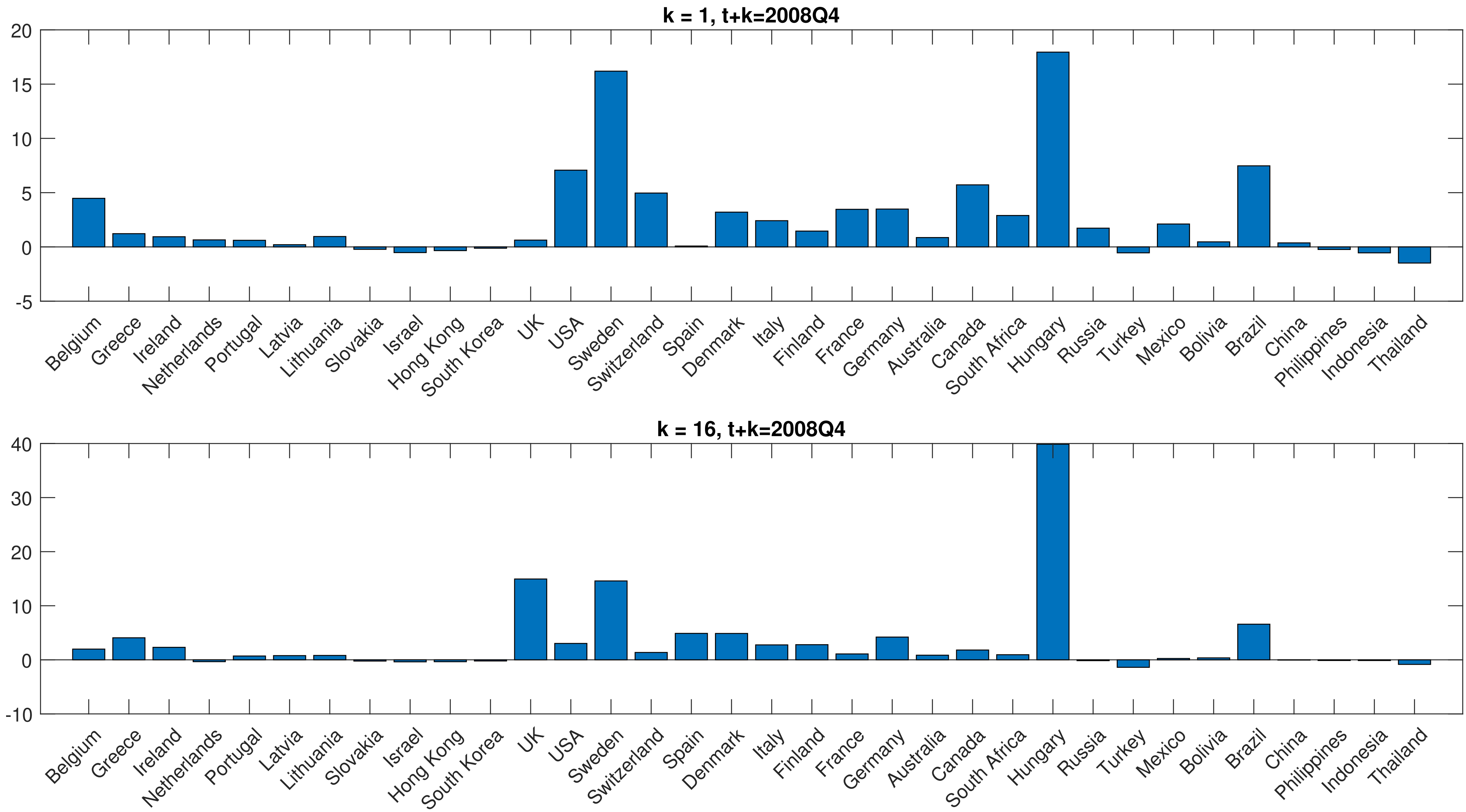


Figure 5: Sums of k-step ahead LPL of output in economy i for UC-FSV relative to UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

5.3 Jointly Forecasting inflation and output growth

With regard to the joint predictive density for inflation and output growth, we first report the sums of joint LPL over time and over economies in Table 7. It is calculated by summing the LPL for the N economies over T_0 to $T-k$ (and for all j, recall that $j=1$ denote inflation, $j=2$ denote output):

$$\text{LPL}_{\cdot,\cdot,\cdot,k} = \sum_{t=T_0}^{t=T-k} \sum_{i=1}^{n} \sum_{j=1}^{2} \log\, p(\widehat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \mathrm{Y}_{1:t}^{(i,j)})$$

The results show that the model, which allows for idiosyncratic stochastic volatility in output growth and cross-country linkages in both inflation and output growth (UC-FSV), provides the most accurate joint forecast for inflation and output growth at all horizons.[17]

Table 7: Sum of k-step-ahead joint log predictive likelihood for 34-economy inflation and output growth

Model	k=4	k=8	k=12	k=16
UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$	0	0	0	0
UC-FSV-$r_y, r_\pi = 0$	679.42	751.62	794.16	615.81
UC-FSV-$r_y = 0$	898.62	1084.28	1084.13	1148.35
UC-FSV	**1513.05**	**1545.20**	**1824.70**	**1672.17**

Next, we study the time-variation in forecast performance to see whether the benefits arise from the forecast during periods of uncertainty. So the second dimension of discussion for joint predictive density for inflation and output growth is sums of joint LPL over time (by summing all j and all economies at time t), which can be calculated by:

$$\text{LPL}_{t,\cdot,\cdot,k} = \sum_{i=1}^{n} \sum_{j=1}^{2} \log\, p(\widehat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \mathrm{Y}_{1:t}^{(i,j)})$$

We plot the results (against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Figure 6. A similar pattern to inflation and output growth was found. To jointly forecast inflation and output growth during periods of uncertainty (like 2008), we find overall good forecast performance under UC-FSV at all horizons. This indicates the importance of taking into account cross-country linkages (in inflation and output growth) for improving forecasts of inflation and output growth, especially during periods of uncertainty.

[17] We do not report the horizon $k=1$ since this has been reported in Table 4. So we refer the reader to Table 4 to see the sum of one-step-ahead joint log predictive likelihood for 34-economy inflation and output growth.

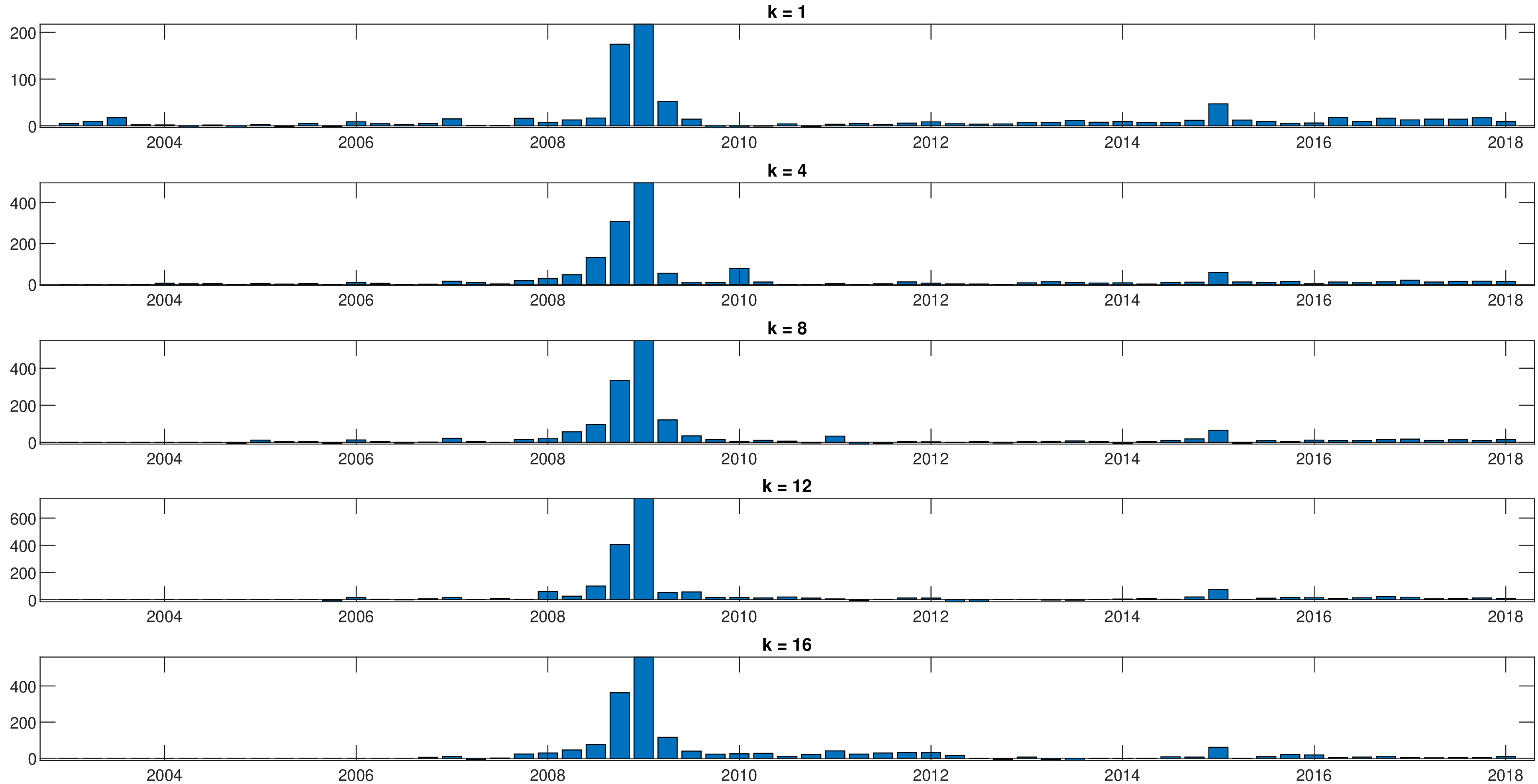


Figure 6: Sums of k-step ahead joint LPL for UC-FSV relative to UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$ over time. The X axis is $t + k$ and represents when to forecast.

Finally, we investigate whether the good forecast performance of periods of uncertainty is driven by particular economies, so the third dimension of discussion for joint predictive density for inflation and output growth is sums of joint LPL at the economy level (by summing all j for economy i), which can be calculated by:

$$\text{LPL}_{t,i,\cdot,k} = \sum_{t=T_0}^{t=T-k} \sum_{j=1}^{2} \log p(\widehat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \text{Y}_{1:t}^{(i,j)})$$

We plot the results (against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Figure 7. A similar pattern to output is found. (This is sensible since the gains in output are much larger than gains in inflation, see Figure 3 and Figure 5). We find overall good forecast performance for UC-FSV for all economies.

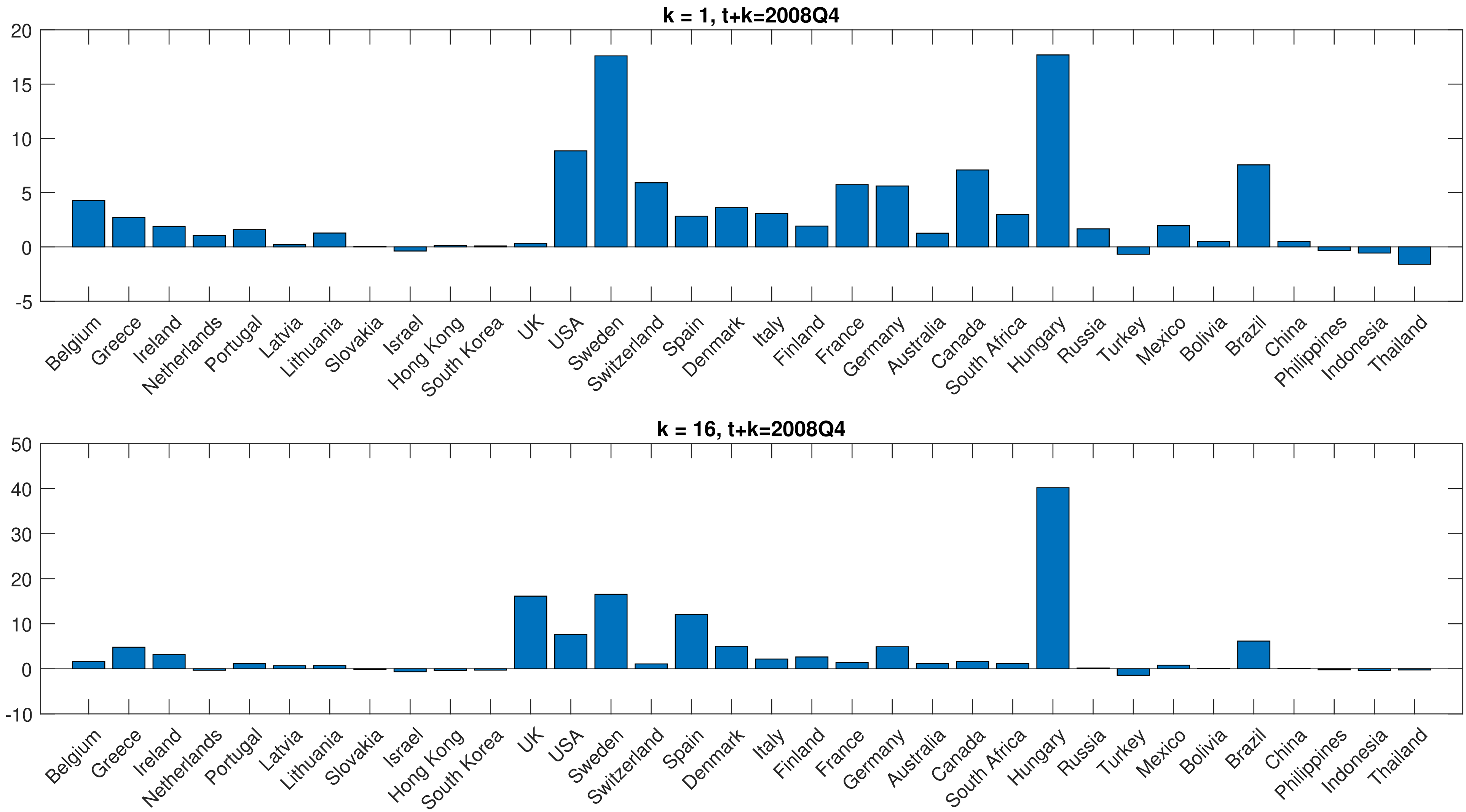


Figure 7: Sums of k-step ahead joint LPL in country i for UC-FSV relative to UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

6 Conclusion

This paper develops a multi-country unobserved components model that allows for cross-country linkages and models economies jointly. The important feature is realized through the factor stochastic volatility. Factor stochastic volatility specification enables us to study the commonality in international macroeconomic uncertainty (global uncertainty). Another important feature of our model is the use of sparsification. We use the sparsification to estimate factor loadings and rely on the post-processing to obtain an estimate for the number of factors. We also use the sparsification to remove stochastic volatility in a data-based manner. Recent research has devoted to speeding up computation and one prominent progress is performing equation-by-equation estimation. The factor stochastic volatility specification also enables us to estimate this high dimensional model equation-by-equation.

In an empirical application we first present evidence of global uncertainty and it coincides with major economic events. Part of the variation captured in the global inflation factor reflects a global business cycle. Finally, we provide a detailed forecasting exercise to evaluate the merits of our model. We find our model can provide more accurate density forecasts, especially if the aim is to forecast periods of uncertainty. And such good forecast performance is for most economies and not driven by particular economies.

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Appendices

A Testing for Time-Variation in Coefficients

In this appendix, we illustrate the method to test for time-variation in coefficients and report the estimated Bayes Factor, which support the constant coefficients model.

What we did is to allow the coefficients in a single-country UC-SV to be time-varying as follows:

$$\pi_{i,t} - \tau^{\pi}_{i,t} = (\rho_{i,0} + \omega^{\rho}_{i}\widetilde{\rho}_{i,t})(\pi_{i,t-1} - \tau^{\pi}_{i,t-1}) + (\alpha_{i,0} + \omega^{\alpha}_{i}\widetilde{\alpha}_{i,t})(y_{i,t} - \tau^{y}_{i,t}) + \varepsilon^{\pi}_{i,t}, \quad \varepsilon^{\pi}_{i,t} \sim \mathcal{N}(0,\ e^{h_{i,t}}) \tag{14}$$

$$y_{i,t} - \tau^{y}_{i,t} = \varphi_{i,1}(y_{i,t-1} - \tau^{y}_{i,t-1}) + \varphi_{i,2}(y_{i,t-2} - \tau^{y}_{i,t-2}) + \varepsilon^{y}_{i,t}, \quad \varepsilon^{y}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{y}) \tag{15}$$

$$\tau^{\pi}_{i,t} = \tau^{\pi}_{i,t-1} + \varepsilon^{\tau\pi}_{i,t}, \quad \varepsilon^{\tau\pi}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{\tau\pi}) \tag{16}$$

$$\tau^{y}_{i,t} = \tau^{y}_{i,t-1} + \varepsilon^{\tau y}_{i,t}, \quad \varepsilon^{\tau y}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{\tau y}) \tag{17}$$

$$h_{i,t} = h_{i,t-1} + \varepsilon^{h}_{i,t}, \quad \varepsilon^{h}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{h}) \tag{18}$$

$$\widetilde{\rho}_{i,t} = \widetilde{\rho}_{i,t-1} + \varepsilon^{\rho}_{i,t}, \quad \varepsilon^{\rho}_{i,t} \sim \mathcal{N}(0,\ 1) \tag{19}$$

$$\widetilde{\alpha}_{i,t} = \widetilde{\alpha}_{i,t-1} + \varepsilon^{\alpha}_{i,t}, \quad \varepsilon^{\alpha}_{i,t} \sim \mathcal{N}(0,\ 1) \tag{20}$$

We assume a normal prior with zero mean and variance ten for $\rho_{i,0}$, ω^{ρ}_{i}, $\alpha_{i,0}$, ω^{α}_{i}. The prior for other parameters are kept the same as UC-SV.

The test for time-variation in $\rho_{i,t}$ ($\alpha_{i,t}$) is equivalent to a test of $\omega^{\rho}_{i} = 0$ ($\omega^{\alpha}_{i} = 0$), we calculate the Bayes factor in favor of the unrestricted model against the restricted version where $\omega^{\rho}_{i} = 0$ as:

$$BF_{\rho_i} = \frac{p(\omega^{\rho}_{i} = 0)}{p(\omega^{\rho}_{i} = 0 |\ y)} \tag{21}$$

So if BF_{ρ_i} is larger than 1, then the Bayes Factor is in favor of the unrestricted model. In this part, the unrestricted model is a time-varying ρ_i. For simplicity, we compare the log Bayes Factor. So a positive log Bayes Factor supports the time-varying coefficient ρ_i. We can calculate the log Bayes Factor for ω^{α}_{i} similarly.

Using the data in the empirical section, we report the log Bayes Factor in Table 8. We find most log Bayes Factors are negative (except for 3 cases: log BF_{ρ_i} for Latvia, Turkey and Mexico), so we think this result strongly supports constant coefficients models.

Table 8: The estimated log Bayes factors for ω_i^ρ and ω_i^α

Economies	log BF_{ρ_i}	log BF_{α_i}
Belgium	-2.83	-1.03
Greece	-2.42	-0.99
Ireland	-2.00	-0.86
Netherlands	-2.21	-2.23
Portugal	-2.61	-1.52
Latvia	0.08	-1.81
Lithuania	-2.20	-2.01
Slovakia	-2.64	-2.33
Israel	-3.30	-3.48
Hong Kong	-2.54	-3.65
South Korea	-1.71	-2.63
UK	-1.44	-3.37
USA	-2.86	-3.63
Sweden	-2.79	-2.28
Switzerland	-2.80	-2.15
Spain	-3.15	-2.54
Denmark	-3.40	-2.45
Italy	-2.30	-2.85
Finland	-2.95	-3.02
France	-2.72	-2.82
Germany	-3.07	-2.94
Australia	-3.13	-3.09
Canada	-3.02	-1.27
South Africa	-1.97	-3.45
Hungary	-3.00	-3.57
Russia	-1.41	-3.37
Turkey	1.08	-3.42
Mexico	2.99	-3.44
Bolivia	-1.02	-3.68
Brazil	-2.18	-3.10
China	-2.09	-2.87
Philippines	-2.99	-2.81
Indonesia	-2.91	-2.39
Thailand	-3.37	-1.41

B Estimates of factor loading matrices

In this appendix, we report the posterior estimates of factor loading matrices under UC-FSV. Basically, we have two classes of factors:

(1): global inflation factor $\boldsymbol{f}_t$, and its loading matrix is L_π, L_π is $N \times r_\pi$, in our empirical application, $N = 34$, $r_\pi = 5$. Table 9 is the loadings of global inflation factor. We report the posterior mean of the five factors' loadings, but only the quantiles of first factorâs loadings for brevity.

(2): global growth factor $\boldsymbol{g}_t$, and its loading matrix is L_y, L_y is $N \times r_y$, in our empirical application, $N = 34$, $r_y = 2$. Table 10 is the loadings of global growth factor. We report the posterior mean and quantiles of the two factors' loadings.

And for identification, we assume the factor loading matrices are lower triangular matrices with ones on the main diagonal, so some elements in L_π and L_y are 1 or 0.

Table 9: Posterior Estimates of factor loading matrix L_π

Economy	1st factor mean	1st factor 16%	1st factor 84%	2nd factor mean	3rd factor mean	4th factor mean	5th factor mean
Belgium	1	1	1	0	0	0	0
Greece	2.29	1.52	3.01	1	0	0	0
Ireland	2.09	1.40	2.74	1.05	1	0	0
Netherlands	1.79	1.17	2.39	-0.97	-0.69	1	0
Portugal	1.57	0.98	2.15	-0.71	-0.17	0.51	1
Latvia	1.96	1.17	2.75	0.66	0.28	0.20	-0.71
Lithuania	2.26	1.41	3.08	1.00	0.35	0.19	-0.82
Slovakia	2.05	1.35	2.72	-0.09	-0.07	0.50	0.23
Israel	2.06	1.31	2.81	-0.29	0.02	-0.12	0.34
Hong Kong	0.93	0.15	1.74	0.60	0.34	-0.90	-0.73
South Korea	1.53	0.99	2.05	-0.18	-0.21	-0.24	-0.63
UK	1.83	1.24	2.38	0.09	-0.13	0.00	-0.22
USA	2.94	2.02	3.81	-0.73	-0.45	0.29	-0.34
Sweden	1.98	1.32	2.61	0.03	0.15	-0.44	-0.59
Switzerland	1.73	1.17	2.26	-0.16	0.07	-0.27	0.34
Spain	3.08	2.11	3.97	-0.03	-0.01	0.71	0.87
Denmark	1.69	1.14	2.21	0.04	0.03	0.21	-0.03
Italy	1.32	0.87	1.74	-0.42	-0.11	0.11	0.75
Finland	1.23	0.77	1.68	0.42	0.38	-0.18	-0.31
France	2.10	1.46	2.68	0.69	0.59	-0.25	0.26
Germany	2.09	1.43	2.68	0.36	0.15	0.46	-0.23
Australia	2.16	1.47	2.80	0.01	0.29	-0.21	0.44
Canada	2.44	1.65	3.19	-0.08	-0.01	-0.54	-0.45
South Africa	1.72	1.01	2.43	-0.79	-0.68	0.45	-0.27
Hungary	3.00	1.86	4.15	0.14	0.11	0.46	-0.05
Russia	0.26	-0.65	1.17	0.48	0.52	-0.57	-0.04
Turkey	3.04	1.79	4.31	0.15	0.13	-0.12	0.04
Mexico	0.49	0.02	0.97	0.41	0.22	0.07	0.18
Bolivia	0.82	-0.04	1.69	0.65	0.25	-0.52	-0.85
Brazil	-0.36	-1.02	0.30	0.17	0.15	-0.50	-0.43
China	0.38	-0.11	0.89	0.50	0.29	-0.62	-0.86
Philippines	1.33	0.68	1.98	-0.25	-0.14	-0.19	-0.12
Indonesia	0.61	0.05	1.18	-0.26	-0.04	-0.16	0.12
Thailand	2.52	1.67	3.33	0.23	-0.04	-0.16	-0.70

Table 10: Posterior Estimates of factor loading matrix L_y

	1st factor			2nd factor		
Economy	mean	16%	84%	mean	16%	84%
Belgium	1	1	1	0	0	0
Greece	2.11	1.35	2.86	1	1	1
Ireland	3.91	2.61	5.21	-2.28	-5.00	0.40
Netherlands	1.75	1.23	2.28	2.61	1.59	3.89
Portugal	1.60	1.14	2.06	1.04	-0.15	2.22
Latvia	1.39	0.26	2.52	0.46	-2.13	3.04
Lithuania	3.70	2.22	5.18	5.54	1.06	10.09
Slovakia	2.99	1.84	4.12	4.63	1.79	7.37
Israel	1.08	0.74	1.41	-1.02	-2.19	0.03
Hong Kong	3.57	2.63	4.51	-0.18	-2.42	1.98
South Korea	2.93	2.20	3.66	-2.03	-4.41	0.21
UK	1.34	0.94	1.74	-1.19	-2.81	0.24
USA	1.60	1.20	1.99	-0.68	-2.00	0.57
Sweden	2.91	2.24	3.57	-0.12	-2.33	1.99
Switzerland	1.74	1.37	2.10	-0.95	-2.30	0.28
Spain	0.70	0.48	0.93	0.12	-0.65	0.84
Denmark	2.12	1.46	2.78	0.40	-1.49	2.26
Italy	2.08	1.62	2.54	-0.23	-1.65	1.05
Finland	3.28	2.28	4.27	4.23	2.38	6.28
France	1.41	1.13	1.70	-0.12	-1.00	0.72
Germany	2.56	1.95	3.18	2.23	0.91	3.72
Australia	0.58	0.23	0.93	-1.62	-2.80	-0.62
Canada	1.34	0.98	1.70	0.81	-0.24	1.87
South Africa	1.04	0.73	1.34	0.60	-0.27	1.49
Hungary	1.31	0.45	2.24	-1.53	-4.37	1.17
Russia	3.36	2.48	4.24	-0.62	-3.09	1.76
Turkey	3.83	2.65	5.00	1.05	-1.50	3.55
Mexico	2.52	1.79	3.26	3.37	2.01	5.04
Bolivia	0.83	0.34	1.33	-0.48	-2.01	0.98
Brazil	3.02	2.20	3.83	-2.27	-4.93	0.22
China	0.95	0.51	1.39	-1.15	-2.63	0.23
Philippines	1.37	0.69	2.05	3.03	1.29	4.94
Indonesia	0.60	0.26	0.94	-0.08	-1.39	1.14
Thailand	2.87	2.12	3.63	0.74	-1.21	2.63

C Estimates of constant coefficients

In this appendix, we report the posterior estimates of constant coefficients: ρ, α, φ_1, φ_2. Before describing the detailed characteristics of each constant coefficient (ρ, α, φ_1, φ_2), we first summarize the relative change of constant coefficients under the multi-country UC-FSV against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$, to assess the effects of contemporaneous cross-country linkages on them.

In Table 11, the number is number of economies. For instance, the "decrease" row "ρ" column is 24, then out of 34 economies, there are 24 economies whose ρ is smaller under the multi-country UC-FSV than the ρ under UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$. ρ is the inflation gap persistence, α is the slope of the Phillips Curve. We find, for most economies, considering global inflation uncertainty will decrease the inflation gap persistence and the slope of the Phillips Curve. Also, we find output gap persistence φ_1 decreases, so allowing for idiosyncratic stochastic volatility in output and global growth uncertainty will decrease the output gap persistence.

Table 11: Relative change under the multi-country UC-FSV against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$

	ρ	α	φ_1	φ_2	$\varphi_1 + \varphi_2$
decrease	24	25	29	9	29
no change	3	8	0	0	0
increase	7	1	5	25	5

Table 12: Posterior estimates of inflation persistence ρ

Economy	UC-FSV mean	UC-FSV 16%	UC-FSV 84%	UC-FSV-$r_y = 0$ posterior mean	UC-FSV-$r_y, r_\pi = 0$ posterior mean	UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$ posterior mean
Belgium	0.28	0.19	0.37	0.28	0.33	0.33
Greece	0.36	0.27	0.45	0.36	0.41	0.41
Ireland	0.46	0.35	0.57	0.46	0.59	0.59
Netherlands	0.29	0.18	0.39	0.29	0.28	0.28
Portugal	0.36	0.27	0.46	0.36	0.47	0.47
Latvia	0.65	0.59	0.71	0.65	0.70	0.70
Lithuania	0.62	0.54	0.70	0.62	0.66	0.66
Slovakia	0.54	0.45	0.62	0.54	0.63	0.63
Israel	0.47	0.38	0.56	0.47	0.53	0.53
Hong Kong	0.56	0.45	0.68	0.57	0.58	0.58
South Korea	0.22	0.12	0.31	0.22	0.31	0.31
UK	0.43	0.34	0.52	0.43	0.41	0.41
USA	0.22	0.14	0.29	0.22	0.28	0.28
Sweden	0.37	0.28	0.45	0.37	0.52	0.52
Switzerland	0.34	0.26	0.42	0.34	0.28	0.28
Spain	0.24	0.17	0.30	0.23	0.39	0.40
Denmark	0.23	0.13	0.32	0.23	0.25	0.25
Italy	0.49	0.41	0.57	0.49	0.59	0.59
Finland	0.49	0.40	0.57	0.49	0.56	0.56
France	0.15	0.09	0.21	0.15	0.26	0.26
Germany	0.07	0.02	0.13	0.08	0.11	0.11
Australia	0.16	0.08	0.24	0.16	0.20	0.20
Canada	0.10	0.03	0.16	0.10	0.11	0.11
South Africa	0.55	0.46	0.65	0.55	0.56	0.56
Hungary	0.40	0.30	0.50	0.40	0.49	0.50
Russia	0.80	0.71	0.89	0.80	0.79	0.79
Turkey	0.93	0.90	0.97	0.94	0.94	0.94
Mexico	0.81	0.75	0.88	0.81	0.80	0.81
Bolivia	0.33	0.23	0.44	0.33	0.32	0.33
Brazil	0.63	0.52	0.74	0.63	0.60	0.61
China	0.53	0.44	0.62	0.52	0.58	0.58
Philippines	0.57	0.47	0.67	0.57	0.61	0.61
Indonesia	0.36	0.26	0.47	0.36	0.35	0.35
Thailand	0.41	0.31	0.51	0.41	0.53	0.53

Table 13: Estimates of slope of Phillips Curve α

Economy	UC-FSV mean	UC-FSV 16%	UC-FSV 84%	UC-FSV-$r_y = 0$ posterior mean	UC-FSV-$r_y, r_\pi = 0$ posterior mean	UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$ posterior mean
Belgium	0.16	0.09	0.23	0.16	0.22	0.22
Greece	0.02	0.00	0.03	0.02	0.02	0.02
Ireland	0.01	0.00	0.02	0.01	0.01	0.01
Netherlands	0.01	0.00	0.03	0.01	0.03	0.03
Portugal	0.05	0.01	0.08	0.05	0.05	0.05
Latvia	0.07	0.04	0.10	0.07	0.10	0.10
Lithuania	0.02	0.00	0.04	0.02	0.04	0.04
Slovakia	0.03	0.01	0.06	0.03	0.08	0.08
Israel	0.07	0.02	0.12	0.07	0.09	0.09
Hong Kong	0.05	0.01	0.09	0.05	0.06	0.06
South Korea	0.03	0.01	0.06	0.04	0.08	0.08
UK	0.02	0.00	0.04	0.02	0.04	0.04
USA	0.05	0.01	0.08	0.05	0.05	0.06
Sweden	0.09	0.06	0.13	0.09	0.14	0.14
Switzerland	0.07	0.03	0.10	0.07	0.15	0.15
Spain	0.08	0.03	0.13	0.07	0.12	0.11
Denmark	0.03	0.01	0.05	0.03	0.05	0.05
Italy	0.04	0.02	0.06	0.04	0.09	0.09
Finland	0.08	0.05	0.10	0.08	0.11	0.11
France	0.02	0.00	0.04	0.02	0.11	0.11
Germany	0.02	0.00	0.03	0.02	0.05	0.05
Australia	0.02	0.00	0.04	0.02	0.03	0.03
Canada	0.10	0.05	0.16	0.10	0.20	0.20
South Africa	0.04	0.01	0.07	0.04	0.06	0.06
Hungary	0.05	0.01	0.10	0.05	0.08	0.08
Russia	0.04	0.01	0.07	0.04	0.04	0.05
Turkey	0.06	0.02	0.11	0.07	0.08	0.09
Mexico	0.04	0.01	0.08	0.04	0.05	0.05
Bolivia	0.09	0.02	0.15	0.09	0.08	0.08
Brazil	0.09	0.03	0.15	0.09	0.08	0.08
China	0.22	0.13	0.30	0.21	0.23	0.22
Philippines	0.02	0.00	0.04	0.02	0.03	0.03
Indonesia	0.15	0.03	0.26	0.14	0.12	0.15
Thailand	0.01	0.00	0.02	0.01	0.02	0.02

Table 14: Estimates of output persistence φ_1

	UC-FSV			UC-FSV-$r_y = 0$	UC-FSV-$r_y, r_\pi = 0$	UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$
Economy	mean	16%	84%	posterior mean	posterior mean	posterior mean
Belgium	0.37	0.27	0.47	0.58	0.58	0.62
Greece	0.09	-0.02	0.20	0.17	0.17	0.10
Ireland	-0.17	-0.29	-0.05	-0.03	-0.03	-0.21
Netherlands	0.14	0.06	0.22	0.31	0.31	0.27
Portugal	0.12	0.02	0.23	0.28	0.28	0.21
Latvia	0.23	0.11	0.34	0.25	0.25	0.17
Lithuania	0.19	0.09	0.28	0.22	0.22	0.06
Slovakia	-0.06	-0.15	0.03	-0.06	-0.06	-0.21
Israel	0.28	0.16	0.39	0.34	0.34	0.12
Hong Kong	0.10	-0.03	0.23	0.39	0.39	0.13
South Korea	-0.06	-0.18	0.06	0.23	0.24	0.27
UK	0.22	0.10	0.35	0.38	0.38	0.53
USA	0.04	-0.07	0.16	0.26	0.26	0.23
Sweden	-0.07	-0.17	0.04	0.07	0.07	0.19
Switzerland	0.21	0.11	0.31	0.46	0.46	0.39
Spain	0.58	0.47	0.71	0.72	0.72	0.73
Denmark	-0.13	-0.24	-0.01	-0.02	-0.02	-0.05
Italy	0.29	0.19	0.39	0.54	0.54	0.44
Finland	0.05	-0.03	0.13	0.16	0.16	0.14
France	0.15	0.06	0.24	0.46	0.45	0.43
Germany	0.06	-0.02	0.14	0.20	0.20	0.25
Australia	-0.08	-0.20	0.03	-0.09	-0.09	-0.20
Canada	0.33	0.23	0.43	0.43	0.42	0.42
South Africa	0.39	0.28	0.50	0.57	0.57	0.39
Hungary	0.20	0.09	0.32	0.24	0.24	0.39
Russia	0.38	0.27	0.48	0.52	0.52	0.43
Turkey	-0.04	-0.14	0.06	0.03	0.03	-0.02
Mexico	0.30	0.22	0.39	0.29	0.29	0.38
Bolivia	-0.18	-0.30	-0.06	-0.17	-0.17	-0.28
Brazil	0.26	0.13	0.38	0.39	0.39	0.13
China	0.08	-0.05	0.22	0.19	0.18	0.07
Philippines	-0.06	-0.16	0.05	-0.02	-0.02	0.00
Indonesia	0.11	-0.03	0.26	0.13	0.14	0.43
Thailand	-0.02	-0.13	0.09	0.10	0.10	-0.10

Table 15: Estimates of output persistence φ_2

	UC-FSV			UC-FSV-$r_y = 0$	UC-FSV-$r_y, r_\pi = 0$	UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$
Economy	mean	16%	84%	posterior mean	posterior mean	posterior mean
Belgium	-0.03	-0.12	0.06	-0.07	-0.07	-0.28
Greece	0.35	0.24	0.45	0.39	0.38	0.35
Ireland	0.10	0.01	0.21	0.15	0.15	-0.05
Netherlands	0.30	0.22	0.37	0.36	0.36	0.11
Portugal	0.27	0.17	0.37	0.33	0.33	0.18
Latvia	0.31	0.21	0.42	0.33	0.33	0.29
Lithuania	0.17	0.08	0.26	0.12	0.12	0.07
Slovakia	0.23	0.15	0.31	0.18	0.18	0.02
Israel	0.04	-0.06	0.13	0.06	0.06	0.08
Hong Kong	0.17	0.08	0.26	0.23	0.23	0.12
South Korea	0.12	0.03	0.21	0.18	0.18	-0.10
UK	0.16	0.05	0.27	0.10	0.09	-0.05
USA	0.22	0.12	0.31	0.15	0.15	0.12
Sweden	0.19	0.10	0.28	0.16	0.16	0.05
Switzerland	0.17	0.09	0.26	0.05	0.05	-0.08
Spain	0.22	0.11	0.32	0.18	0.18	-0.01
Denmark	0.07	-0.04	0.17	0.12	0.12	0.04
Italy	0.08	-0.01	0.16	0.01	0.01	-0.04
Finland	0.19	0.11	0.27	0.24	0.24	0.04
France	0.24	0.16	0.32	0.21	0.21	0.03
Germany	0.16	0.09	0.23	0.09	0.09	-0.05
Australia	0.10	-0.01	0.20	0.10	0.10	-0.04
Canada	0.04	-0.05	0.14	-0.02	-0.02	-0.18
South Africa	0.18	0.07	0.28	0.05	0.05	0.02
Hungary	0.16	0.06	0.25	0.12	0.13	-0.06
Russia	0.05	-0.04	0.15	0.07	0.07	-0.11
Turkey	0.06	-0.03	0.14	0.11	0.11	0.06
Mexico	-0.06	-0.14	0.02	0.03	0.03	-0.23
Bolivia	-0.13	-0.24	-0.01	-0.15	-0.15	-0.24
Brazil	0.14	0.03	0.24	0.08	0.08	-0.05
China	0.05	-0.05	0.16	0.10	0.09	0.01
Philippines	0.09	-0.01	0.18	0.09	0.10	-0.06
Indonesia	-0.01	-0.12	0.10	-0.05	-0.05	-0.11
Thailand	-0.05	-0.14	0.04	0.00	-0.01	0.08

D Testing for Time-Variation in volatilities

In this appendix, we report the estimated log Bayes factors to test for time-variation in volatilities.

Table 16: The estimated log Bayes factors for ω_i^h

Economies	log BF_{h_i} for inflation	log BF_{h_i} for output
Belgium	-6.16	-5.17
Greece	-3.97	6.68
Ireland	-6.52	20.92
Netherlands	-2.86	-8.56
Portugal	-7.86	-5.37
Latvia	-4.63	4.32
Lithuania	-2.48	4.27
Slovakia	82.45	60.82
Israel	21.29	19.80
Hong Kong	-5.52	22.35
South Korea	42.60	75.68
UK	-10.58	-4.79
USA	3.11	-5.79
Sweden	-6.42	-3.43
Switzerland	-4.95	-5.51
Spain	-6.13	-3.94
Denmark	-4.92	-4.83
Italy	-5.64	5.33
Finland	-5.30	-5.11
France	-5.58	-6.11
Germany	-1.93	-3.85
Australia	32.10	-2.67
Canada	-5.03	-6.21
South Africa	-1.29	-4.89
Hungary	3.17	-2.85
Russia	131.11	17.81
Turkey	20.91	3.71
Mexico	-4.99	-6.03
Bolivia	-2.02	-4.72
Brazil	6.08	-2.82
China	-4.20	69.65
Philippines	-3.86	-0.19
Indonesia	151.82	200.64
Thailand	8.09	64.03

E Estimates under the multi-country UC-FSV

In this subsection, we compare the estimates of latent states produced from the multi-country UC-FSV and UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$. Specifically:

1) Estimates of trend inflation τ^π;

2) Estimates of trend growth τ^y;

3) Idiosyncratic inflation uncertainty $\exp(h_t^\pi/2)$ (In fact, $\exp(h_t^\pi/2)$ is the standard deviation, so what we compare is the standard deviation);

4) idiosyncratic growth uncertainty $\exp(h_t^y/2)$.

Estimates of Trend inflation In Figure 8, we report the posterior estimates of trend inflation under the four competing models. The title of each sub-figure is the economy name, followed by the official inflation targets (point target or target bands). For instance, the title of the first sub-figure is "Belgium (2) ", then the first sub-figure depicts the estimate of trend inflation for Belgium and the official inflation target set by Belgium central bank is 2%. Each sub-figure plots the posterior estimates (mean, 16% and 84% quantiles) of trend inflation under the multi-country UC-FSV along with the posterior mean of trend inflation under three competing models. The solid blue lines are the means, 16% and 84% quantiles under UC-FSV, the dotted red lines are posterior means under UC-FSV-$r_y = 0$, the dashed black lines are posterior means under UC-FSV-$r_y, r_\pi = 0$, while the dashed green lines are posterior means under UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

The solid blue lines and the dotted red lines represent the estimate under the models considering global inflation uncertainty in inflation gap equation, while the dashed black lines and the dashed green lines represent the estimate under the models without global inflation uncertainty in inflation gap equation. The first 23 economies are AEs (from Belgium to Canada), followed by 11 EMEs. A pattern which emerges from the results is that considering global inflation uncertainty tend to influence the estimated trend inflation more in AEs than in EMEs. The posterior means under the four competing models are almost coincident in EMEs. However, global inflation uncertainty does generate some differences in AEs, such as Netherlands, USA, Switzerland, Denmark, Italy, France, Germany, Canada. And we observe that such differences indicate that trend inflation is driven by both domestic factors and global factors. For instance, many papers without global inflation uncertainty document that trend inflation for USA has been below 2% since 2012, and this is also observed under our competing models UC-FSV-$r_y, r_\pi = 0$ and UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$, in contrast, the mean estimate under model with global inflation uncertainty decreases to a higher level in 2010. Then it begins to increase, rather than decreasing until 2015.

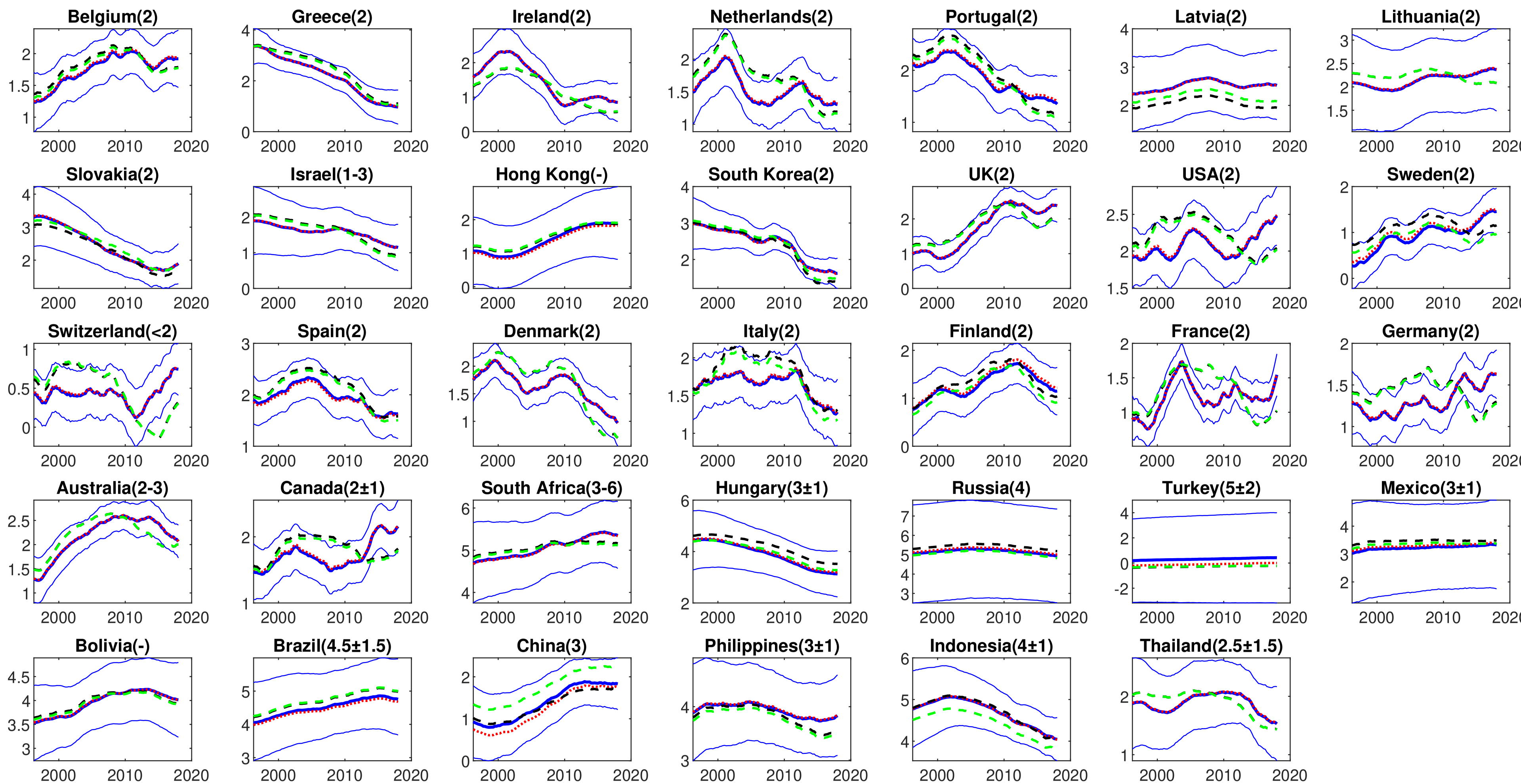

Figure 8: Posterior estimates for trend inflation τ^{π}. The title of each sub-figure is the economy name, followed by the official inflation targets (point target or target bands). For Hong Kong and Bolivia, we do not find the official inflation targets, so we use "-". The solid blue lines are the means, 16% and 84% quantiles under the multi-country UC-FSV, the dotted red lines are posterior means under the multi-country UC-FSV-$r_y = 0$, the dashed black lines are posterior means under UC-FSV-$r_y, r_\pi = 0$, while the dashed green lines are posterior means under UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

Estimates of trend growth In Figure 9, we report the posterior estimates of trend growth under the four competing models. The title of each sub-figure is the economy name. Each sub-figure plots the posterior estimates (mean, 16% and 84% quantiles) of trend growth under the multi-country UC-FSV along with the posterior mean of trend growth under three competing models. The meaning of each line is the same as that in Figure 8. The solid blue lines are the means, 16% and 84% quantiles under the multi-country UC-FSV, the dotted red lines are posterior means under UC-FSV-$r_y = 0$, the dashed black lines are posterior means under UC-FSV-$r_y, r_\pi = 0$, while the dashed green lines are posterior means under UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

We have two modifications in the growth gap equation: allowing for idiosyncratic growth uncertainty (UC-FSV-$r_y, r_\pi = 0$ and UC-FSV-$r_y = 0$) and considering global growth uncertainty (the multi-country UC-FSV).

We first analyze the effect of idiosyncratic growth uncertainty on the estimate of trend growth. This is done through comparing the estimates under UC-FSV-$r_y, r_\pi = 0$ (dashed black lines) with the estimate under UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$ (dashed greed lines), where the error in growth gap equation remains homoscedastic. We find allowing for idiosyncratic growth uncertainty will provide higher estimate of trend growth in many economies (roughly 20 out of 34 economies).

Then, we analyze the effect of global growth uncertainty on the estimate of trend growth. This is done through comparing the estimates under the multi-country UC-FSV (solid blue lines) with the estimate under UC-FSV-$r_y = 0$ (dotted red lines). Similar to the finding in the effect of global inflation uncertainty, we find that considering global growth uncertainty tend to influence the estimated trend inflation more in AEs than in EMEs.

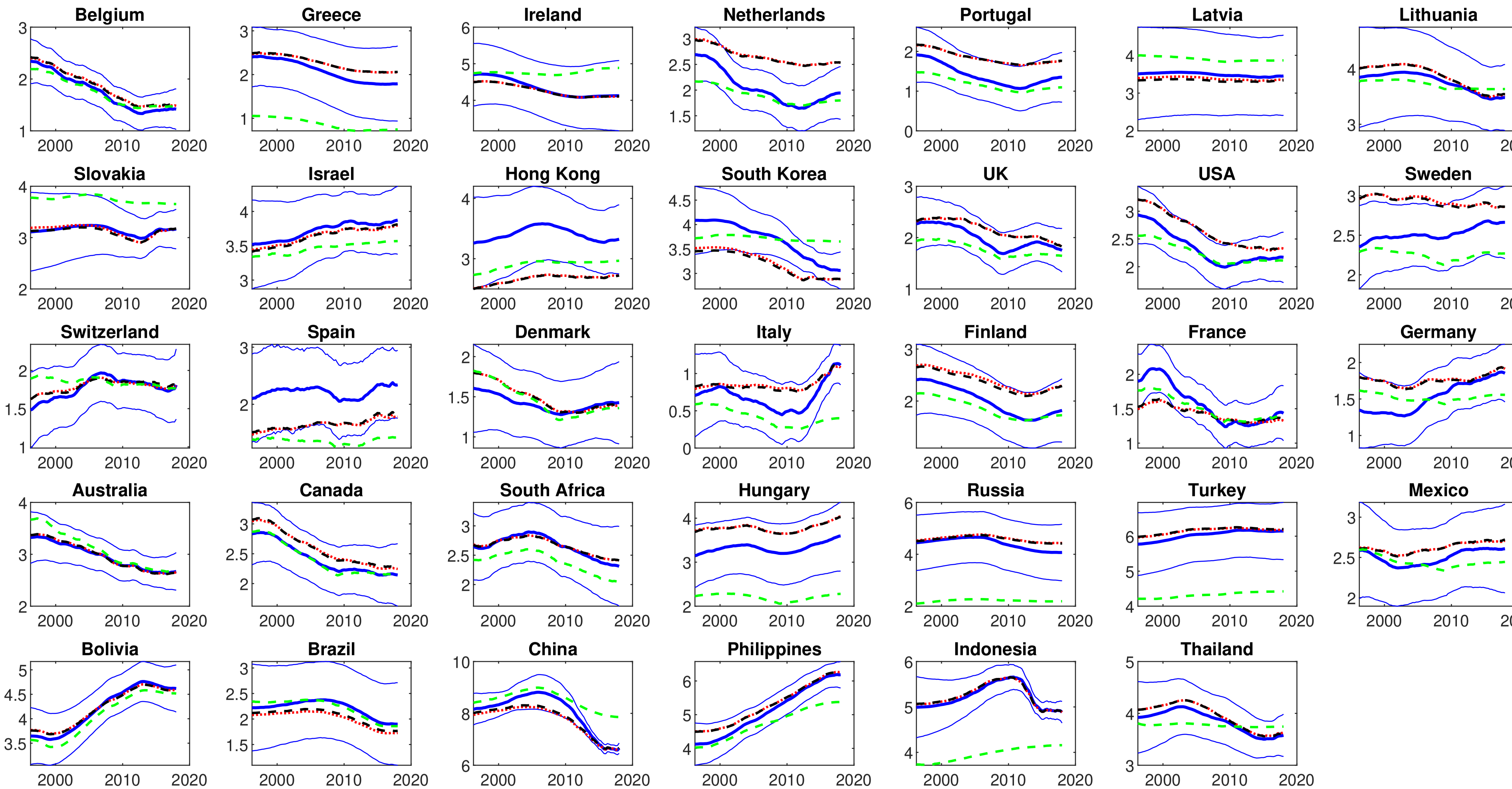

Figure 9: Posterior estimates for trend growth τ^y. The solid blue lines are the means, 16% and 84% quantiles under the multi-country UC-FSV, the dotted red lines are posterior means under UC-FSV-$r_y = 0$, the dashed black lines are posterior means under UC-FSV-$r_y, r_\pi = 0$, while the dashed green lines are posterior means under UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

Estimates of idiosyncratic inflation uncertainty In Figure 10, we report the idiosyncratic inflation uncertainty estimates (i.e., the standard deviation of the shocks to the inflation gap, $\exp(h_t^{\pi}/2)$) under the four competing models. The title of each sub-figure is the economy name. Each sub-figure plots the posterior estimates (mean, 16% and 84% quantiles) under the multi-country UC-FSV along with the posterior mean under three competing models. The meaning of each line is the same as that in Figure 10. The solid blue lines are the means, 16% and 84% quantiles under the multi-country UC-FSV, the dotted red lines are posterior means under UC-FSV-$r_y = 0$, the dashed black lines are posterior means under UC-FSV-$r_y, r_\pi = 0$, while the dashed green lines are posterior means under UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

The solid blue lines and the dotted red lines represent the estimate under the models allowing for cross-country linkages in inflation gap equation, while the dashed black lines and the dashed green lines represent the estimate under the models without cross-country linkages in inflation gap equation. A quick visual inspection shows that allowing for cross-country linkages reduces the spike of idiosyncratic inflation uncertainty. This can be regarded as an evidence supporting that there exist factors driving strong co-movement of inflation across economies. In addition, the idiosyncratic inflation uncertainty in several economies becomes quite flat after allowing for cross-country linkages, suggesting that the uncertainty in their inflation gap equation is driven by global inflation uncertainty, rather than idiosyncratic inflation uncertainty.

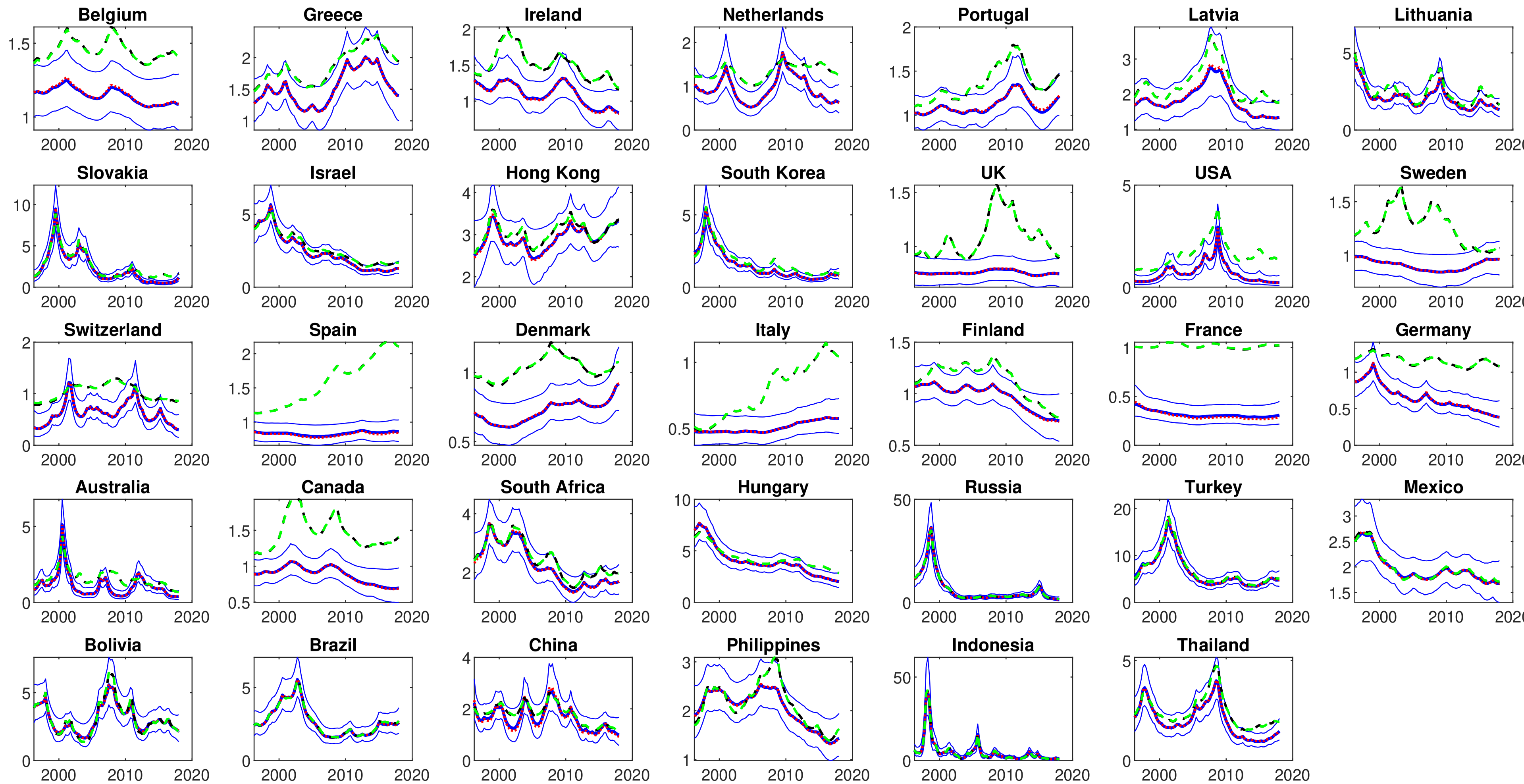

Figure 10: Posterior estimates for idiosyncratic inflation uncertainty $\exp(h_t^\pi/2)$. The solid blue lines are the means, 16% and 84% quantiles under the multi-country UC-FSV, the dotted red lines are posterior means under UC-FSV-$r_y = 0$, the dashed black lines are posterior means under UC-FSV-$r_y, r_\pi = 0$, while the dashed green lines are posterior means under UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

Estimates of idiosyncratic growth uncertainty In Figure 11, we report the idiosyncratic growth uncertainty estimates (i.e., the standard deviation of the shocks to the output gap, $\exp(h_t^y/2)$) under the four competing models. The title and the meaning of each line is the same as that in Figure 10.

The solid blue lines and the dotted red lines represent the estimate under the models allowing for cross-country linkages in growth gap equation, while the dashed black lines and the dashed green lines represent the estimate under the models without cross-country linkages in growth gap equation. The pattern found for idiosyncratic inflation uncertainty can also be found for idiosyncratic growth uncertainty. We again observe that allowing for cross-country linkages reduces the spike of idiosyncratic growth uncertainty. This indicates that there exist factors driving strong co-movement of output across economies. The idiosyncratic growth uncertainty in many economies becomes quite flat after allowing for cross-country linkages, suggesting that the uncertainty in their growth gap equation is driven by global growth uncertainty, rather than idiosyncratic growth uncertainty. This number of idiosyncratic growth uncertainty becoming flat is higher than the number of idiosyncratic inflation uncertainty becoming flat, which provides evidence for papers assuming that the error in growth gap equation is homoscedastic, but at the same time supports the need to allow for cross-country linkages.

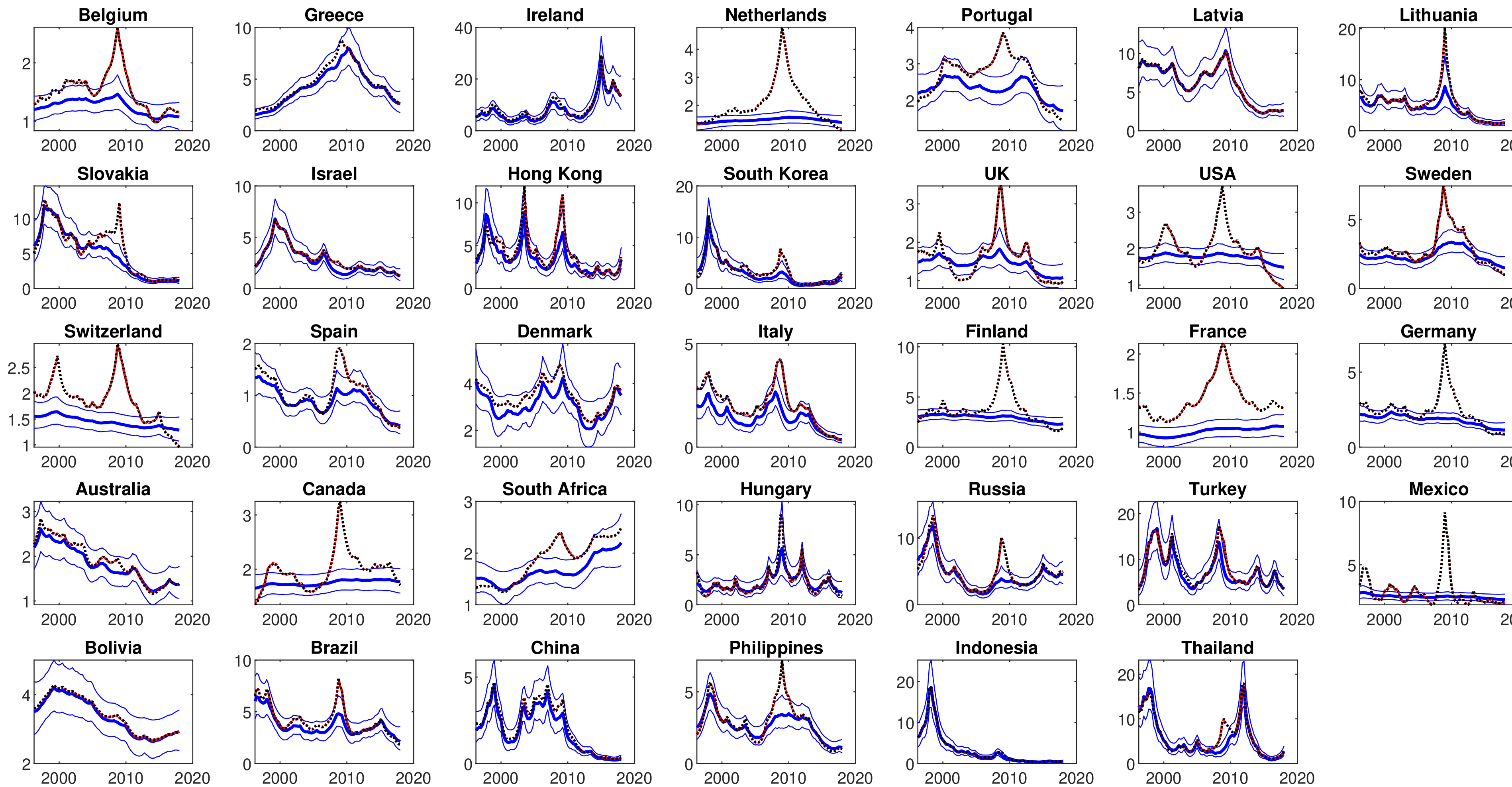

Figure 11: Posterior estimates for idiosyncratic growth uncertainty $\exp(h_t^y/2)$. The solid blue lines are the means, 16% and 84% quantiles under the multi-country UC-FSV, the dotted red lines are posterior means under UC-FSV-$r_y = 0$, the dashed black lines are posterior means under UC-FSV-$r_y, r_\pi = 0$.

F Sparsification: To remove stochastic volatility

We assess whether the Horseshoe prior can successfully shrink strongly the parameter space (ω^h) but at the same time provides enough flexibility to allow for non-zero elements if necessary.

We first test for time-variation in the volatility of inflation and output, then plot the estimated time-varying standard deviation ($\exp(h_{j,t}/2)$) to see whether it coincides with the test result. Of course, to test for time-variation in the volatility, a gold standard is using marginal likelihood (Bayes Factor is the ratio of two marginal likelihoods). However, in our settings where we allow for time-variation in volatility, the computation of marginal likelihood requires integrating out all the states, making it a nontrivial task. Therefore, we follow the method developed in Chan (2018). More specifically, since we notice that the model without SV is a restricted version of the model with SV, the Bayes Factor can be calculated using the Savage-Dickey density ratio, thus avoiding the computation of marginal likelihood. The Bayes Factor in favor of the unrestricted model (model with SV) can be obtained using the Savage-Dickey density ratio as

$$\mathrm{BF}_{h_j} = \frac{p(\omega_j^h = 0)}{p(\omega_j^h = 0|\ y)}$$

So if BF_{h_j} is larger than 1, then the Bayes Factor is in favor of the unrestricted model. In this part, the unrestricted model is time-varying h_j. For simplicity, we compare the log Bayes Factor. So a positive log Bayes Factor supports a time-varying h_j.

On the computation of posterior density ($p(\omega_j^h = 0|\ y)$), we can obtain the posterior distribution given output from MCMC algorithm, then it is direct to compute the posterior density. On the computation of prior density ($p(\omega_j^h = 0)$), since we use the Horseshoe prior on ω_j^h, $p(\omega_j^h = 0)$ does not have a convenient analytical form. But, given the hyperparameters ($\lambda_j^{\omega^h}, \tau^{\omega^h}, \nu_j^{\omega^h}, \xi^{\omega^h}$) in Horseshoe prior, $p(\omega_j^h = 0|\ \lambda_j^{\omega^h}, \tau^{\omega^h}, \nu_j^{\omega^h}, \xi^{\omega^h})$ is Normal. Thus, if we have output from a prior simulator, we can approximate $p(\omega_j^h = 0)$ by

$$\widehat{p}(\omega_j^h = 0) = \frac{1}{S}\sum_{s=1}^{S} p(\omega_j^h = 0|\ \lambda_j^{\omega^{h,s}}, \tau^{\omega^{h,s}}, \nu_j^{\omega^{h,s}}, \xi^{\omega^{h,s}})$$

This approximation applies for any prior which has a hierarchical form. The estimated log Bayes Factor is reported in Appendix D. For time-variation in the volatility of inflation in 34 economies, 12 economies are in favor of time-variation in the volatility (their log Bayes Factor are positive). For time-variation in the volatility of output in 34 economies, 14 economies are in favor of time-variation in the volatility.

To see whether the estimated time-varying standard deviation coincides with the estimated log Bayes Factor, we report the two in Figure 12 and Figure 13. Figure 12 depicts the estimated time-varying standard deviation for inflation gap equations. The title of each sub-figure is the economy name, followed by the estimated log Bayes Factor. For instance, the title of the first sub-figure is "Belgium (-6.16) ", then the first sub-figure depicts the estimated time-varying standard deviation for Belgium inflation and the the estimated log Bayes Factor is -6.16, which is negative and implies that the log Bayes Factor does not supports time-variation in the volatility of Belgium inflation. Figure 13 depicts the estimated time-varying standard deviation for growth gap equations. The title is named in the same way as inflation. The first sub-figure depicts the estimated time-varying standard deviation for Belgium growth and the the estimated log Bayes Factor is -5.17, which is negative and implies that the log Bayes Factor does not support time-variation in the volatility of Belgium growth.

We find that the estimates of both the log Bayes Factor and the time-varying standard deviation are sensible and coincide with past research. For USA, the log Bayes Factor supports time-varying volatility of inflation, while does not support time-varying volatility of growth. This is consistent with what we observe from the estimated time-varying standard deviation for inflation and growth. We observe marked spike in Figure 12, while it remains quite flat in Figure 13. Zaman (2022), Kabundi et al. (2021), among many others, assume that the error in the growth gap equation remains homoscedastic. The consistency among the log Bayes Factor, the time-varying standard deviation and past literature implies that the Horseshoe prior can successfully remove unimportant small SV and at the same time provides enough flexibility to allow for SV if necessary.

In addition, we find that, for several economies, the log Bayes Factor supports time-varying volatility of growth and we also observe marked spikes from the time-varying standard deviation. This result points towards a big advantage of our proposed model, which allows for SV in growth gap equation. While past research assume the error in growth gap equation is homoscedastic, such assumption displays a tendency to be over-restricted in multi-country study and ignores patterns observed under the model allowing for SV in growth gap equations. Omitting the SV can severely affect the reliability of the estimates of the trend growth.

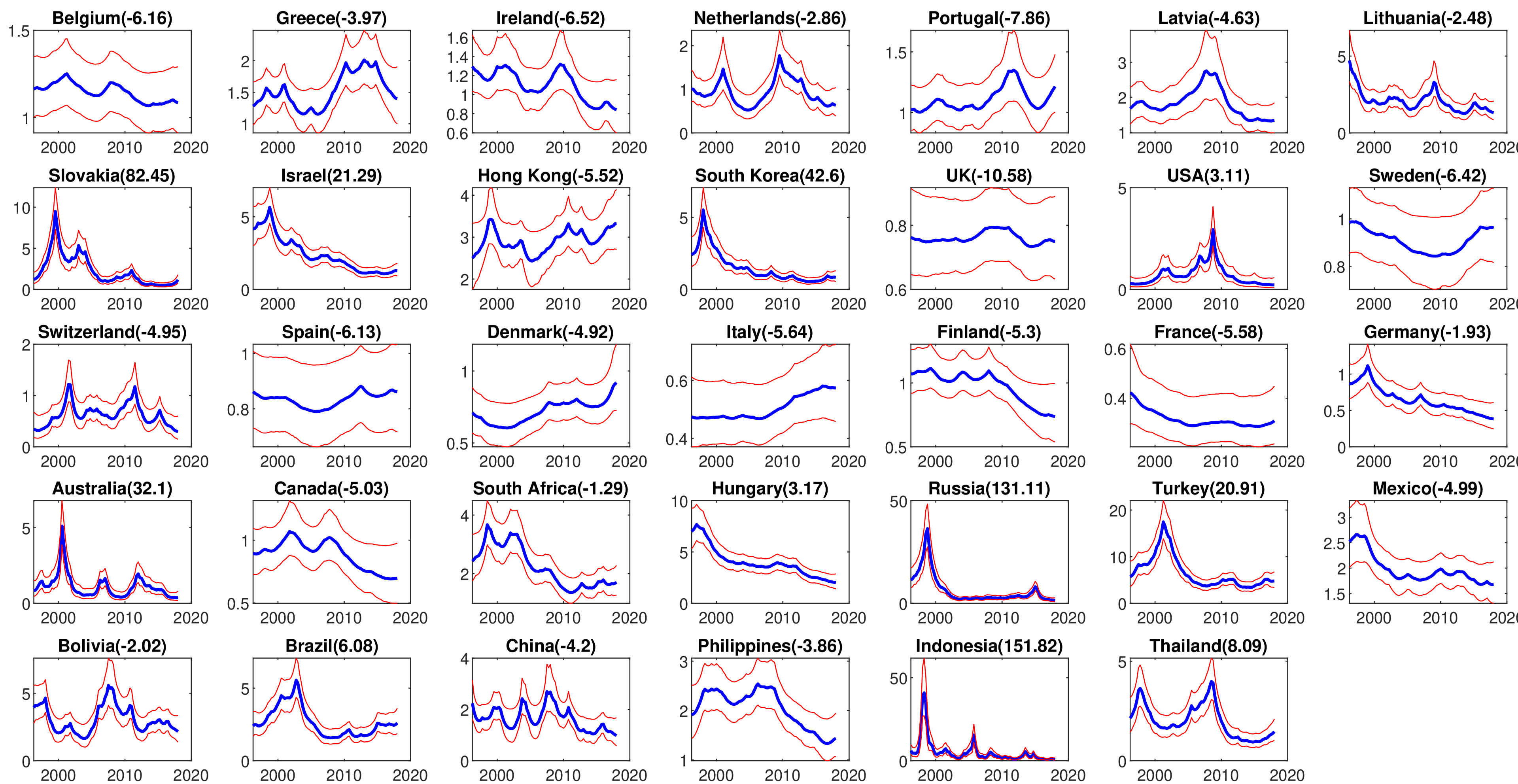

Figure 12: The estimated log Bayes Factor and time-varying standard deviation in inflation gap equations. The solid blue lines are the means, 16% and 84% quantiles of time-varying standard deviation.

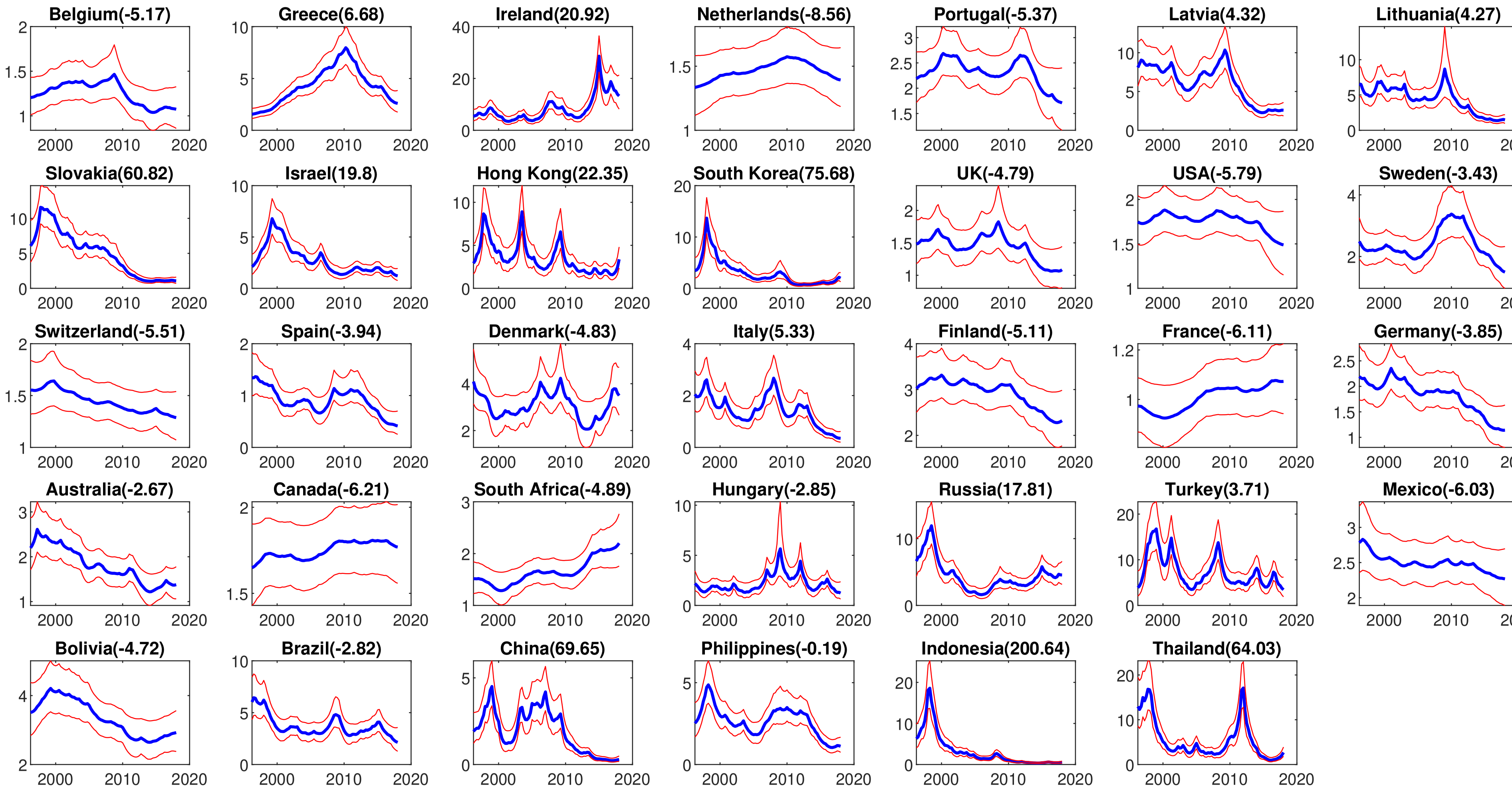

Figure 13: The estimated log Bayes Factor and time-varying standard deviation in growth gap equations. The solid blue lines are the means, 16% and 84% quantiles of time-varying standard deviation.

G Factor loading matrices: unconstrained

In the main paper, we assume that the factor loading matrices L_π and L_y are both a lower triangular matrix with ones on the main diagonal. Here we drop this constraint and assume they are full. The prior is shown in Eq. (9). Then we experiment two cases:

(1) shrink the time-invariant part of log-volatility $h_{j,0}$ using the Horseshoe prior (this is what we use in the main paper);

(2) not shrink the time-invariant part of log-volatility, and use a normal prior with zero mean and variance one.

We post-process the posterior draws to obtain an estimate of the number of factors. Table 17 reports the estimate in case (1), and Table 18 reports the estimate in case (2). We find a consistent conclusion that there is no global factor in inflation gap equations (there is no singular value that is larger than the threshold), and there is one global factor in growth gap equations.

Table 17: Posterior number of factors (shrink $h_{j,0}$)

Singular values (Descending)	The inflation equation threshold = 118.35	The output equation threshold = 106.10
First	50.47	121.83
Second	25.96	55.88
Third	12.68	45.12

Table 18: Posterior number of factors (not shrink $h_{j,0}$)

Singular values (Descending)	The inflation equation threshold = 117.30	The output equation threshold = 105.00
First	49.52	123.20
Second	24.20	44.62
Third	7.27	16.41

H The Horseshoe prior

In this appendix, we show the inverse-Gamma representation of the Horseshoe prior. In particular, we consider

$$\begin{aligned}
\lambda_j^\beta \mid \nu_j^\beta &\sim \mathcal{IG}\left(\frac{1}{2}, \frac{1}{\nu_j^\beta}\right), \\
\tau^\beta \mid \xi_\beta &\sim \mathcal{IG}\left(\frac{1}{2}, \frac{1}{\xi_\beta}\right), \\
\nu_j^\beta &\sim \mathcal{IG}\left(\frac{1}{2}, 1\right), \\
\xi_\beta &\sim \mathcal{IG}\left(\frac{1}{2}, 1\right),
\end{aligned}$$

where $\left(\nu_1, \ldots, \nu_{k_\beta}\right)$ and ξ are independent auxiliary random variables. This representation has been used in Cross et al. (2020).