1 Dynamic Factor Model

Part (b) to (d), we consider three different model specifications of a DFM. Using the stata output, we define the three models below. First we define the vector of dependent variables $\mathbf{y}_t = (rpi_t, ip_t, emp_t, sp_t)'$ where rpi_t is the real personal income, ip_t is industrial production, emp_t is employment and sp_t is stock prices.

1.1 Model 1 $p = q = m = 1$

$$\mathbf{y}_t = \mathbf{c} + \mathbf{P}\mathbf{f}_t + \mathbf{u}_t, \tag{1}$$

$$\mathbf{f}_t = \mathbf{A}_1\mathbf{f}_{t-1} + \nu_t, \nu_t \sim N(0, \sigma_f^2), \tag{2}$$

$$\mathbf{u}_t = \mathbf{C}_{1t}\mathbf{u}_t + \epsilon_t, \epsilon_t \sim N(0, \Sigma), \tag{3}$$

Using the stata output, we estimate: $\mathbf{c} = \begin{bmatrix} 0.003 \\ 0.002 \\ 0.001 \\ 0.005 \end{bmatrix}$, $\mathbf{P} = \begin{bmatrix} 0.001 \\ 0.003 \\ 0.001 \\ 0.001 \end{bmatrix}$, $\mathbf{A}_1 = 0.8$, $\mathbf{C}_{1t} = \begin{bmatrix} -0.18 & 0 & 0 & 0 \\ 0 & 0.13 & 0 & 0 \\ 0 & 0 & -0.50 & 0 \\ 0 & 0 & 0 & 0.24 \end{bmatrix}$,

and $\Sigma = \begin{bmatrix} 0.00003 & 0 & 0 & 0 \\ 0 & 0.00004 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.011 \end{bmatrix}$. Figure 1 plots smoothed estimates of $\mathbf{f}_t$.

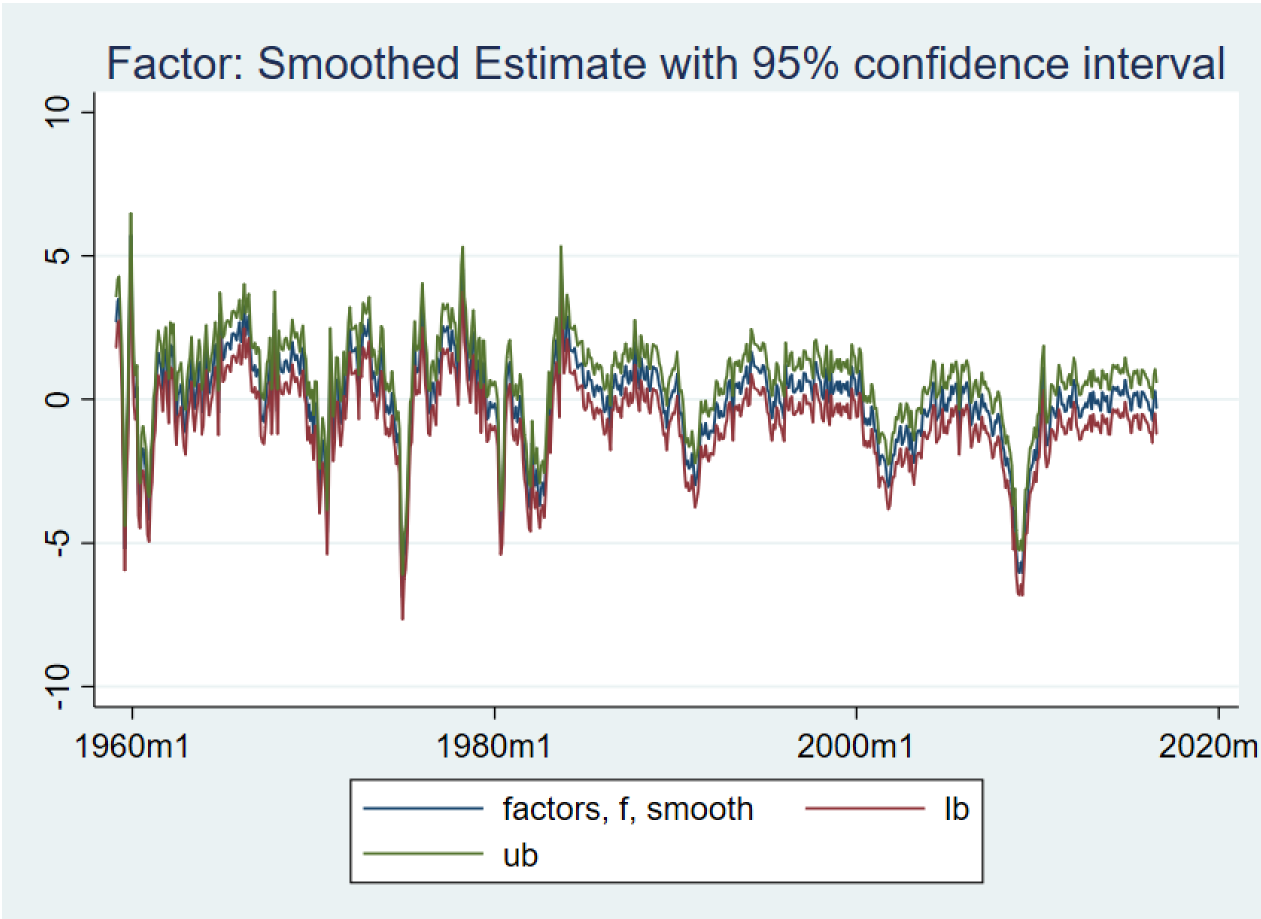

Figure 1: Plot of the smoothed estimates of $\mathbf{f}_t$ with 95% confidence interval

1.2 Model 2 $p = 0, q = m = 1$

$$\mathbf{y}_t = \mathbf{c} + \mathbf{P}\mathbf{f}_t + \mathbf{u}_t, \tag{4}$$

$$\mathbf{u}_t = \mathbf{C}_{1t}\mathbf{u}_t + \epsilon_t, \epsilon_t \sim N(0, \Sigma), \tag{5}$$

Using the stata output, we estimate: $\mathbf{c} = \begin{bmatrix} 0.003 \\ 0.002 \\ 0.001 \\ 0.005 \end{bmatrix}$, $\mathbf{P} = \begin{bmatrix} 0.002 \\ 0.005 \\ 0.002 \\ 0.002 \end{bmatrix}$, $\mathbf{C}_{1t} = \begin{bmatrix} -0.16 & 0 & 0 & 0 \\ 0 & -0.03 & 0 & 0 \\ 0 & 0 & 0.95 & 0 \\ 0 & 0 & 0 & 0.24 \end{bmatrix}$,and $\Sigma = \begin{bmatrix} 0.00003 & 0 & 0 & 0 \\ 0 & 0.00004 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.011 \end{bmatrix}$. Figure 2 plots smoothed estimates of $\mathbf{f}_t$.

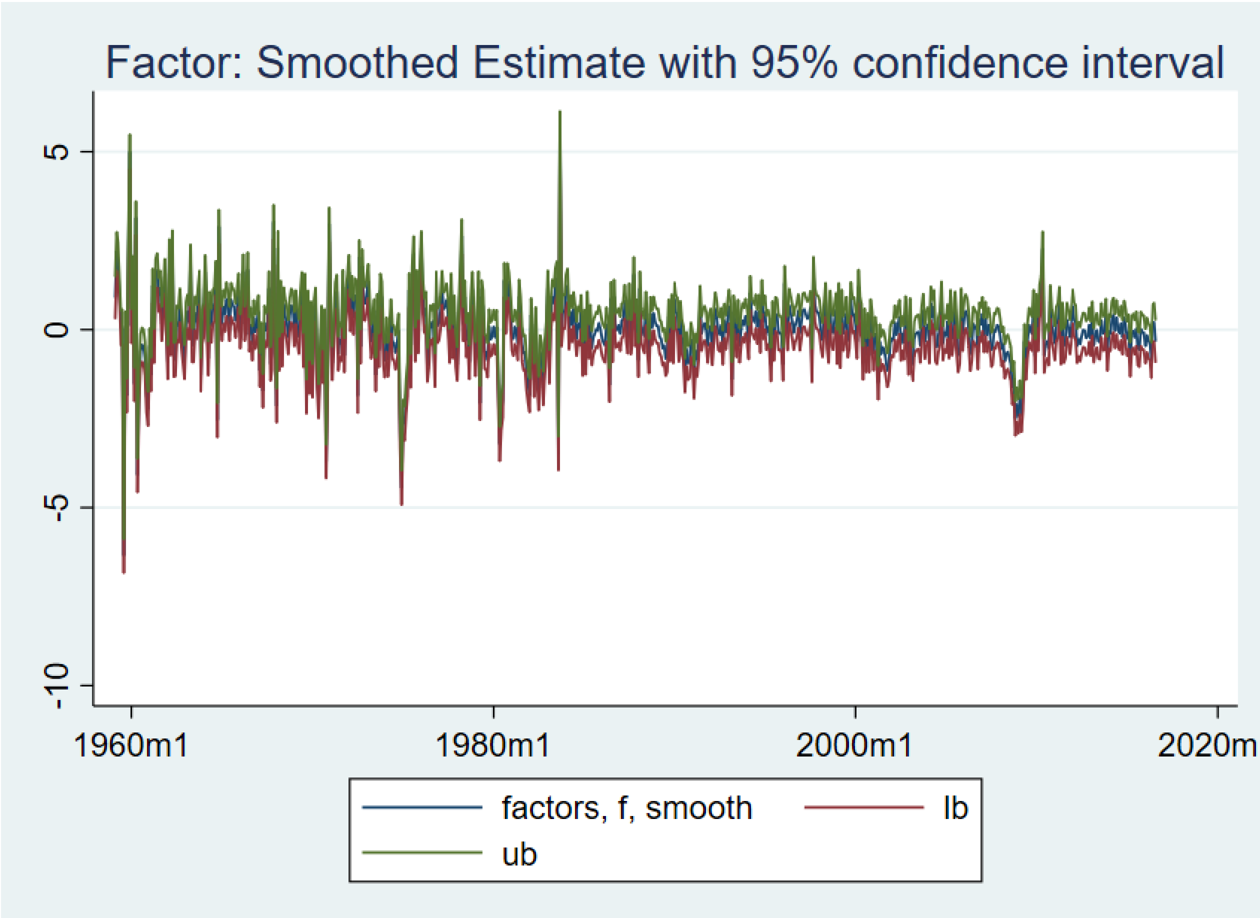

Figure 2: Plot of the smoothed estimates of $\mathbf{f}_t$ with 95% confidence interval

1.3 Model 3 $p = q = 0, m = 1$

$$\mathbf{y}_t = \mathbf{c} + \mathbf{P}\mathbf{f}_t + \mathbf{u}_t, \tag{6}$$

$$\mathbf{u}_t = \epsilon_t, \epsilon_t \sim N(0, \Sigma), \tag{7}$$

Using the stata output, we estimate: $\mathbf{c} = \begin{bmatrix} 0.003 \\ 0.002 \\ 0.001 \\ 0.005 \end{bmatrix}$, $\mathbf{P} = \begin{bmatrix} -0.002 \\ -0.006 \\ -0.002 \\ -0.002 \end{bmatrix}$, and $\Sigma = \begin{bmatrix} 0.00003 & 0 & 0 & 0 \\ 0 & 0.00003 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.012 \end{bmatrix}$.

Figure 3 plots smoothed estimates of $\mathbf{f}_t$.

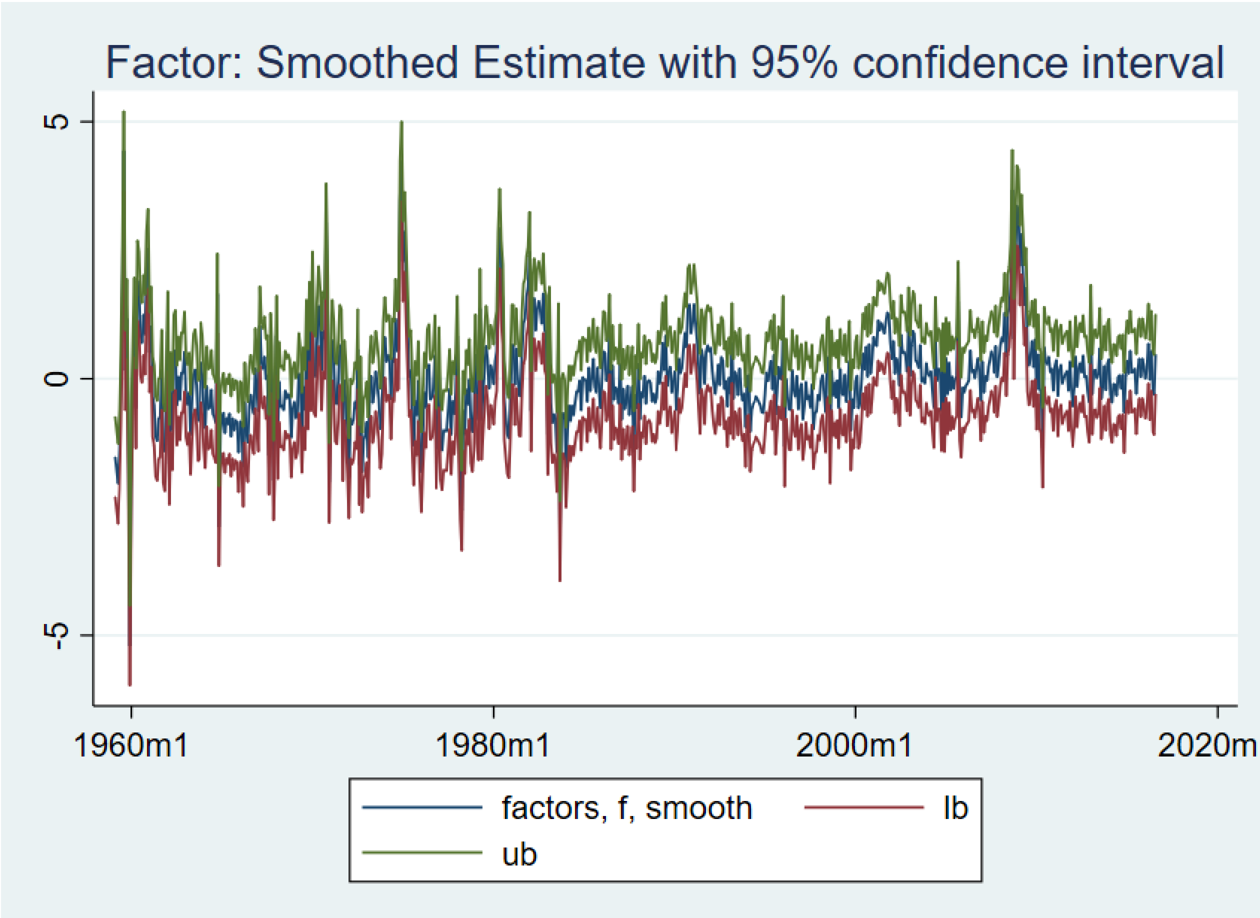

Figure 3: Plot of the smoothed estimates of $\mathbf{f}_t$ with 95% confidence interval

2 Model Comparison

	AIC	BIC
Model 1	**-19819.24**	**-19742.09**
Model 2	-19673.34	-19600.73
Model 3	-19391.13	-19336.67

Table 1: Model Comparison

Clearly from Table 1, Model 1 is the preferred model as it has the lowest AIC and BIC measures out of the three models.