1 Markov Switching AR model

Let's define the Markov-Switching AR(4) model that we want to estimate:

$$y_t = c_{s_t} + \phi_{1,s_t} y_{t-1} + \phi_{2,s_t} y_{t-2} + \phi_{3,s_t} y_{t-3} + \phi_{4,s_t} y_{t-4} + \epsilon_{t,s_t} \sim N(0, \sigma^2_{s_t}), \tag{1}$$

where y_t is defined to be real GDP. $s_t \in [1,2]$ and the transition probabilities are defined as $\Pr(s_{t+1} = j | s_t = i) = p_{ij}$, which is defined as the probability from transitioning from regime i to j. The plot of real GDP over time is in figure 1.

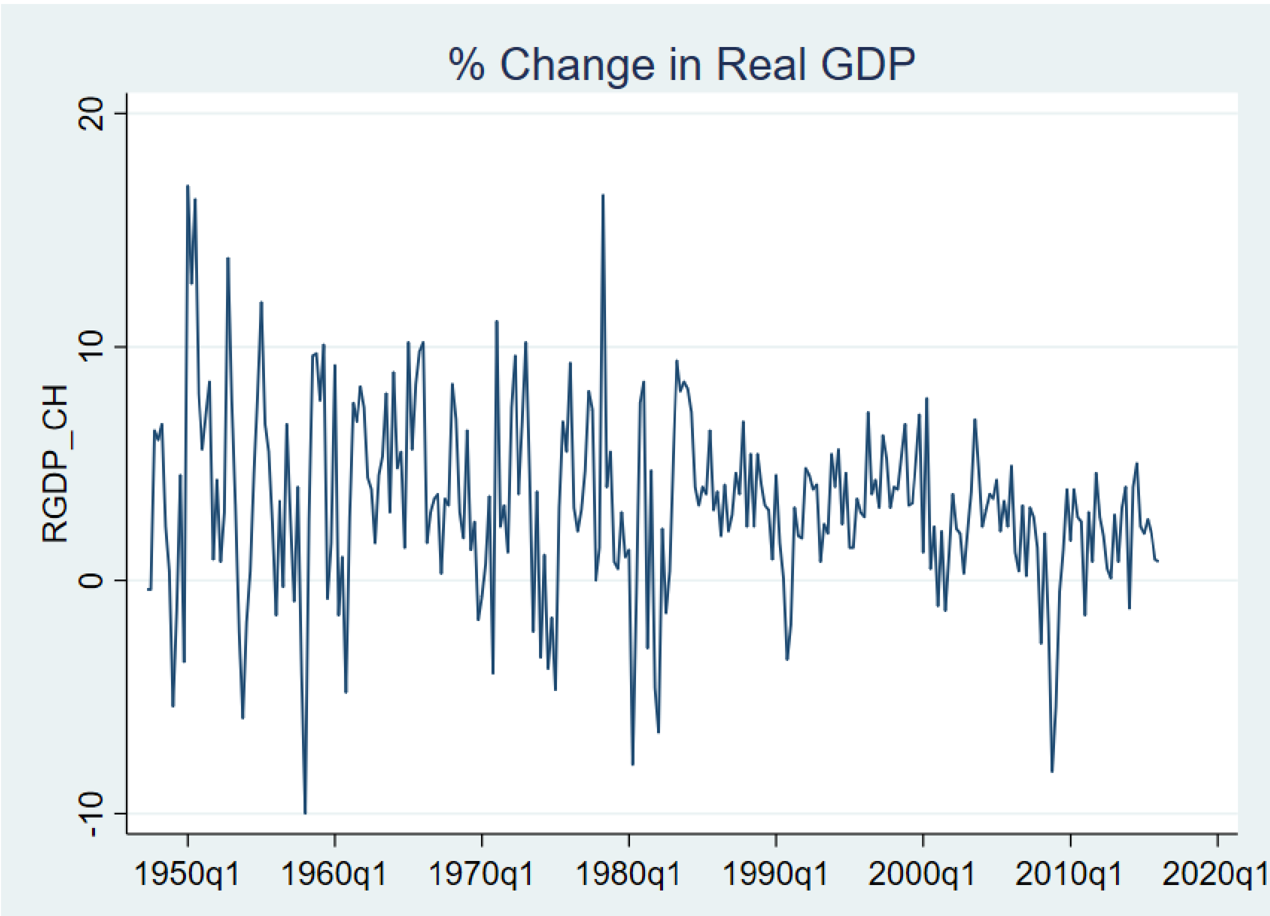

Figure 1: Plot of Real GDP over time

The estimated stata output is in the dofile file "MSAR4". Thus, the reported estimated coefficients are:

$$y_t = \begin{cases} 1.66 + .21y_{t-1} + .27y_{t-2} - .1y_{t-3} + .06y_{t-4} + \epsilon_{t,1} \sim N(0, 1.88) & \text{if } s_t = 1, \\ 2.43 + .35y_{t-1} + .11y_{t-2} - .06y_{t-3} - .11y_{t-4} + \epsilon_{t,2} \sim N(0, 4.52) & \text{if } s_t = 2, \end{cases}$$

The matrix of the transition probabilities are

$$\mathbf{P} = \begin{bmatrix} 0.98 & 0.02 \\ 0.01 & 0.99 \end{bmatrix},$$

There is a high probability that once the model enters a particular state, it is likely they will remain at that particular state. The expected duration of s_1 is about 66 quarters and for s_2 it is about 75 quarters. Figure 2 plots each state's smoothed probabilities over time and from the graph it appears that US real GDP was on average in the regime 2 from 1950-85 and in regime 1 from the 1990s onwards. Potentially this regime switch could be due to the introduction of inflation targeting by the Fed during the early 1990s.

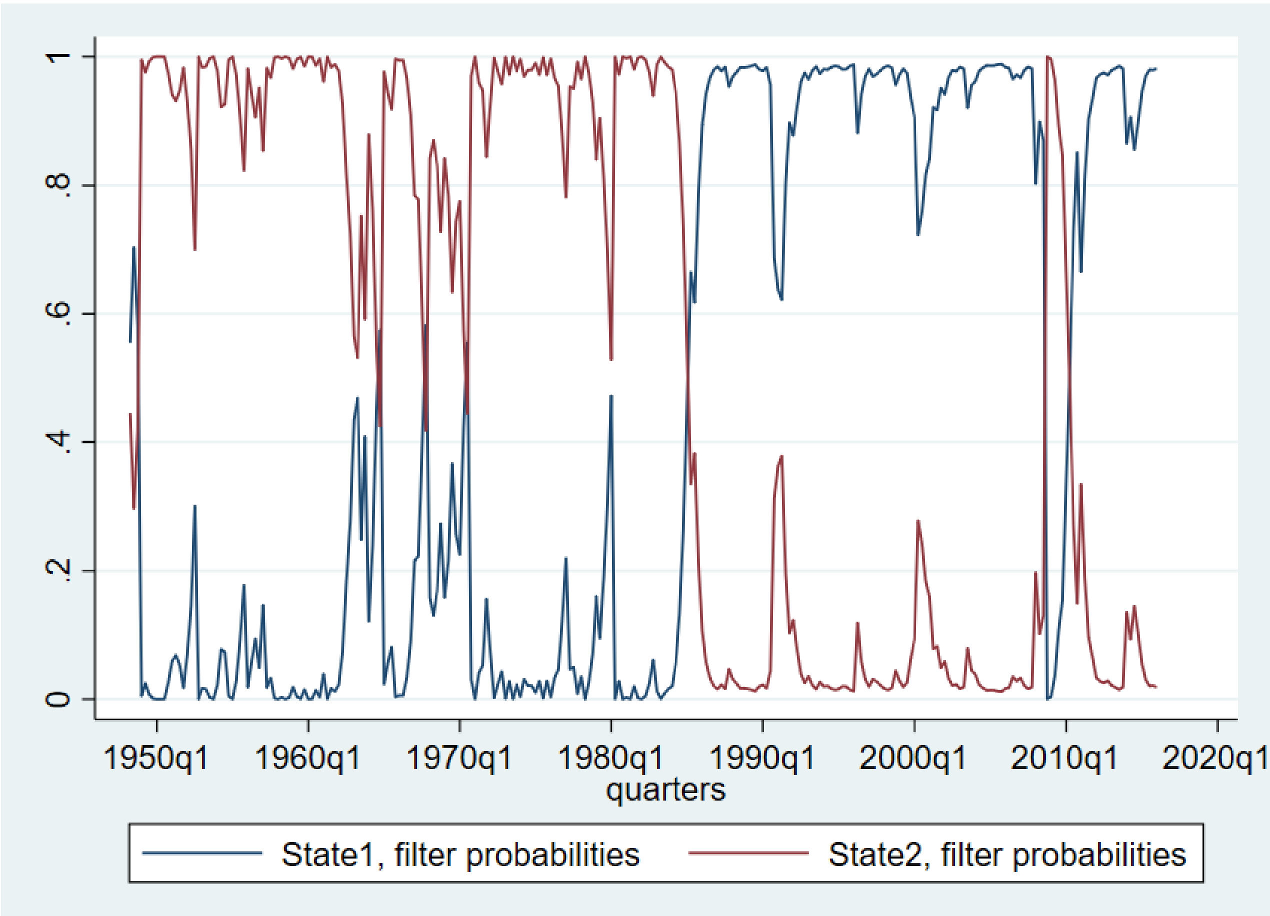

Figure 2: Plot of the State's probabilities over time from MSAR(4)

Now let's consider a Markov-Switching AR(2) model. From the stata estimated output, we have

$$y_t = \begin{cases} 1.66 + .17y_{t-1} + .26y_{t-2} + \epsilon_{t,1} \sim N(0, 1.88) & \text{if } s_t = 1, \\ 2.04 + .36y_{t-1} + .06y_{t-2} + \epsilon_{t,2} \sim N(0, 4.55) & \text{if } s_t = 2, \end{cases}$$

The matrix of the transition probabilities are the same as before

$$\mathbf{P} = \begin{bmatrix} 0.98 & 0.02 \\ 0.01 & 0.99 \end{bmatrix},$$

The expected duration of s_1 is about 67 quarters and for s_2 it is about 80 quarters. Figure 3 plots each state's smoothed probabilities over time and they are very much the same as before.

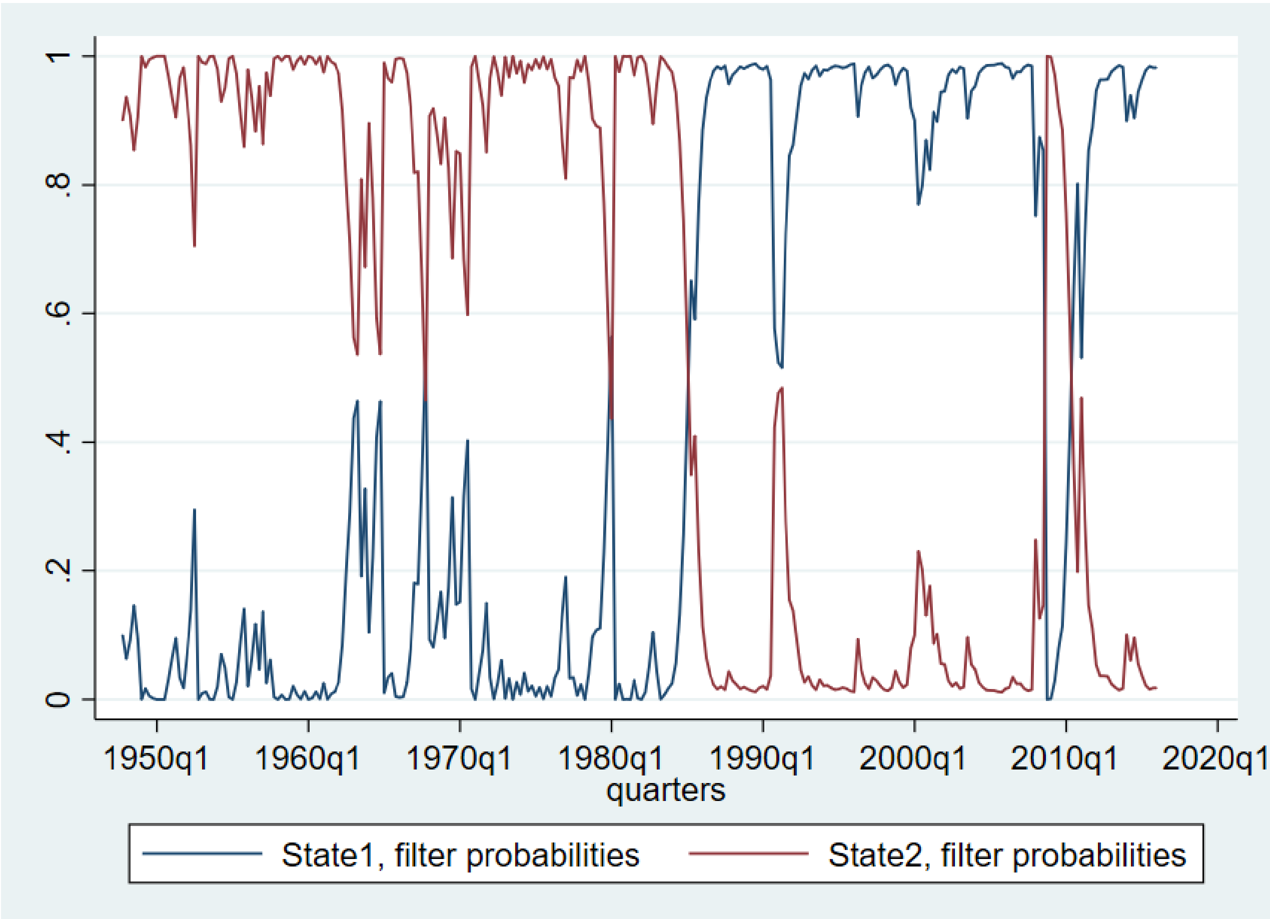

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Figure 3: Plot of the State's probabilities over time from MSAR(2)

Table 1 reports the AIC and BIC values for both models, and both measures report conflicting results. For example, based on the AIC, the MSAR(4) is the preferred model, but in the BIC, the MSAR(2) is the preferred the model. Note that the BIC has a larger penalty term compared to the AIC and as a result, tend to favour more parsimonious models. Thus, it appears that the BIC is heavily penalising the two additional lag coefficients in the MSAR(4) model.

	AIC	BIC
MSAR(4)	**1427.98**	1478.46
MSAR(2)	1435.37	**1471.51**

Table 1: Model Comparison