Computer Tutorial 1: State Space Models

The file inflation_data.xls contains data on several measures of inflation (along with several inflation forecasts). In the example in the lectures I used CPI inflation and you may wish to use this in all of the questions. But if you wish you can experiment with alternative measures of inflation when answering the questions below.

The do file I provide uses Stata's state space command, sspace. Stata also has another command, ucm (for "unobserved components model" which covers many of what the lectures called "structural time series" models). Stata describes ucm as an easy-to-use implementation of sspace (it is menu-driven and does not require a do file). I would prefer you to use the more general sspace command since it can be used for many more models than ucm. And ucm cannot estimate every model asked for in this problem set. But you may wish to experiment with ucm for some of the models in this question.

Question 1: In my lecture slides on state space models I went through an empirical application using inflation data and estimating trend inflation using models of unmoored and anshored inflation expectations (see my lecture slides for complete details). Both models involved a measurement equation of the form:

$$y_t = \alpha_t + \varepsilon_t$$

where α_t is trend inflation. The unmoored model has a state equation of the form:

$$\alpha_{t+1} = \alpha_t + u_t$$

and the anchored model has one of the form:

$$\alpha_{t+1} = c + \rho\alpha_t + u_t.$$

My Stata do file which carries out the empirical application is provided (Question1.do). The intention of this question is simply to get you to understand the format of Stata's state space command and understand the output it produces. Look through the do file, run it and examine the output it produces. While you are doing this, keep in mind the following facts and questions. Stata estimates general Normal linear state space models. The models given in this question are restricted versions of this general model. How can you impose the necessary restrictions in Stata? What are the parameters of each model and how does Stata report their estimates? How can you decide whether any parameter is significantly different from zero?

Question 2: The AR(1) model is often written in "deviations from mean" form as:

$$y_t - \mu = \rho\left(y_{t-1} - \mu\right) + \varepsilon_t,$$

where it can be shown that $E\left(y_t\right) = \mu$. This form of the model is convenient since researchers are often interested in having a direct estimate of the unconditional mean of the series and this is not possible in the conventional way of writing the AR(1) model. That is, in the conventional AR(1) model:

$$y_t = \alpha + \rho y_{t-1} + \varepsilon_t$$

the mean is

$$E\left(y_t\right) = \frac{\alpha}{1-\rho}.$$

The AR(1) model in deviations from mean form can be written as a state space model:

$$\begin{aligned} y_t &= \mu + u_t \\ u_t &= \rho u_{t-1} + \varepsilon_t \end{aligned}$$

where the first equation is the measurement equation and the second equation is a state equation involving the state u_t.

Modify the Stata do file from Question 1 to allow for state space estimation of this model. Use Stata's ARIMA command to estimate the conventional AR(1) model. Compare the estimates of $E\left(y_t\right)$ produced by the two variants of the AR(1) model.

Question 3: The local linear trend model extends the local level model as follows:

$$y_t = \alpha_t + \varepsilon_t$$

with α_t being interpreted as trend inflation but its state equation takes the form

$$\alpha_{t+1} = \alpha_t + \beta_t + u_{1t}$$

with additional state equation:

$$\beta_{t+1} = \beta_t + u_{2t}.$$

i) Adapt the do file of Question 1 to estimate the local linear trend model.
ii) Compare filtered and smoothed estimates of trend inflation from the local linear trend and local level models.
iii) Use an information criterion to decide between the local level and local linear trend models.
iv) Researchers sometimes find that the local linear trend model produces an estimate of trend inflation which is too erratic. Have you found this? In such cases, a common recommendation is to set $\sigma^2_\beta = 0$ (where $\sigma^2_\beta = var\left(u_{2t}\right)$). Modify your do file to do this. Is the resulting estimate of trend inflation smoother than before?