## Dynamic Factor Models

- Stata estimates DFMs of the form:

$$\begin{aligned} y_t &= Pf_t + Qx_t + u_t \\ f_t &= Rw_t + A_1 f_{t-1} + .. + A_p f_{t-p} + v_t \\ u_t &= C_1 u_{t-1} + .. + C_q u_{t-q} + \varepsilon_t \end{aligned}$$

- $\varepsilon_t$ is i.i.d. $N(0, \Sigma_\varepsilon)$
- $v_t$ is i.i.d. $N(0, \Sigma_v)$
- $y_t$ is $N \times 1$ vector of dependent variables
- $f_t$ is $m \times 1$ vector of factors
- $x_t$ and $w_t$ are $n_x$ exogenous variables
- $P, Q, R, A_1, .., A_p, C_1, .., C_q$ are all matrices of parameters to be estimated
- $P$ are the factor loadings
- Note $x_t$ and/or $w_t$ could contain intercept

Notation: Equations

- Equation 1 ($y_t$ equation) contains N "subequations" since there are N dependent variables.
- Equation 2 ($f_t$ equation) contains m "subequations" since there are m factors.
- Equation 3 ($u_t$ equation) contains N "subequations" since there are N dependent variables each of which has a corresponding error term.

Answer to Question

- e.varname (an *element* of the vector $u_t$) stands for the error in the equation for varname ( i.e an *element* of the vector $y_t$)
- So var(e.varname) gives $var(u_t)$
- Throughout, we are referring to the errors in Equation 1, $u_t$.
- When we have the static factor model (p=q=0) only the Equations 1 and 2 disappear respectively.

Example

- ld_income is the income variable which has been log differenced.
- ld_income is contained in the vector of dependent variables, $y_t$.
- Thus e.ld_income is the error term in the equation for ld_income.
- So var(e.ld_income) gives the variance of the error term in the equation for ld_income.

Interpretation

- In the static factor model, where we have only Equation 1, the errors, $u_t$, reflect the idiosyncratic component or variable specific factor (i.e. the bit that the factors, $f_t$, which are included in each $y_t$ "subequation" can't explain).
- So var(e.ld_income) gives the variance of the idiosyncratic error/variable specific factor.
- Relative to other things, I don't think this aspect is too important but if pushed I would comment on the magnitude of the variance of the idiosyncratic error/variable specific factor and the statistical significance (by examining the p-value).